2次元におけるある準線形ケラー・シーゲル・ナヴィエ・ストークス系の弱解の大域存在と有界性

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1. Problem and Known Results

The chemotaxis-Navier-Stokes system with position dependent sensitivity is the mathematical model which is combined with the system in Tuval et al. [**2**] and the one in Xue-Othmer [**5**]. This mathematical model describes the dynamics of swimming bacteria which live in fluid (like Escherichia coli) and its positive chemotaxis. One of the interesting points of this system is it has the position dependent sensitivity. This means that the bacteria do not move to the gradient of the chemical substance directly but with a rotation.

This talk deals with the the following degenerate system:

$$
\text{(KSNS)} \qquad \begin{cases} n_t = \Delta n^m - \nabla\cdot(S(n,c,x)n\nabla c) - u\cdot\nabla n, & x\in\Omega,\ t>0, \\ c_t = \Delta c - nc - u\cdot\nabla c, & x\in\Omega,\ t>0, \\ u_t = \Delta u - \kappa(u\cdot\nabla)u - \nabla P + n\nabla\phi, & x\in\Omega,\ t>0, \\ \nabla\cdot u = 0, & x\in\Omega,\ t>0, \\ (\nabla u - S(n,c,x)n\nabla c)\cdot\nu = \nabla c\cdot\nu = 0,\ u=0, & x\in\partial\Omega,\ t>0, \\ n(x,0)=n_0(x), \quad c(x,0)=c_0(x), \quad u(x,0)=u_0(x), & x\in\Omega, \end{cases}
$$

where $\Omega\subset\mathbb{R}^2$ is a bounded domain with smooth boundary $\partial\Omega$, $m>1$, $\kappa=0$ or $\kappa=1$, $\nabla\phi\in\big(L^\infty(\Omega)\big)^2$. The notation ν is the outward normal unit vector to $\partial\Omega$ and $S(n,c,x)$ is a square matrix of order 2. The initial data are $n_0, c_0\ge 0$ and $\nabla\cdot u_0=0$ on Ω.

Here the unknown function (n,c,u,P) represents the cell density, the chemical concentration, the fluid velocity field and the associated pressure. The rotation $S(n,c,x)$ models the position dependent sensitivity.

We focus on the case that (KSNS) with $m>1, \kappa=1$ under the following condition on $S(n,c,x)=(S_{ij}(n,c,x))_{i,j\in\{1,2\}}$.

Condition on *S*.

(A1) $\qquad S_{ij}\in C^1([0,\infty)\times[0,\infty)\times\overline{\Omega})$ for $i,j\in\{1,2\}$,

(A2) $\qquad |S(u,v,x)|\le\widetilde{s}(v) \quad$ for all $(u,v,x)\in[0,\infty)\times[0,\infty)\times\overline{\Omega}$,

where $\widetilde{s}$ is a non-decreasing function on $[0,\infty)$.

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Let us state the definition of global weak solutions to (KSNS).

Definition of weak solutions. A quadruple of functions (n, c, u, P) defined on $\Omega \times [0, \infty)$ is called *a global weak solution* to (KSNS) if

- $n \in L^\infty(\Omega \times (0, \infty)),\ \nabla u^m \in L^2_{\rm loc}([0, \infty); L^2(\Omega)),\ n \geq 0,$
- $c \in L^\infty(\Omega \times (0, \infty)) \cap L^\infty(0, \infty; W^{1,q}(\Omega))\ (\forall q \in [2, \infty)),\ c \geq 0,$
- $u \in \left(L^\infty(0, \infty; H^1_0(\Omega))\right)^2$, $\Delta u \in \left(L^2_{\rm loc}([0, \infty); L^2(\Omega))\right)^2$,
- (n, c, u, P) satisfies (KSNS) in the sense of distributions.

Known results with $\boldsymbol{\kappa = 1}$. We mainly deal with the case that $\kappa = 1$. In this case, Winkler ([**3, 4**]) proved global existence and the large time behavior of (KSNS) with $m = 1$ and $S = I$ (identity matrix). However, there is no result for (KSNS) with $m > 1$ and with general S. Thus our main purpose is to show global solvability and boundedness in (KSNS).

2. Main Result

Now we present the main theorem.

Theorem ([**1**])

Let $m > 1$, $\kappa = 1$ and $(n_0, c_0, u_0) \in L^\infty(\Omega) \times W^{1,\infty}(\Omega) \times D(A^\alpha)$ with some $\alpha \in (\frac{1}{2}, 1)$, where A is the Stokes operator in $L^2_\sigma(\Omega) = \{\varphi \in L^2; \nabla \cdot \varphi = 0\}$. Assume that S satisfies (A1), (A2). Then there exists at least one global weak solution to (KSNS) with

$$\|n\|_{L^\infty(0,\infty;L^\infty(\Omega))} \leq K,$$

where $K > 0$ is a constant that depends only on the initial data and Ω.

Remark. In the case that $\kappa = 0$, we can also obtain the same result more easily.

References

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