Stability of Gaussian bounds for heat kernels of Schrödinger operators on Riemannian manifolds

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Let M be a complete, non-compact Riemannian manifold. Let $d(x,y)$ be the geodesic distance and m the Riemannian volume. We suppose that the heat kernel $p(t,x,y)$ associated with the Laplace-Beltrami operator Δ satisfies global Gaussian lower and upper bounds (Li-Yau estimate): for every $x,y \in M$ and $t > 0$,

$$\frac{C_1 \exp\left(-c_1 \frac{d^2(x,y)}{t}\right)}{m(B(x,\sqrt{t}))} \leq p(t,x,y) \leq \frac{C_2 \exp\left(-c_2 \frac{d^2(x,y)}{t}\right)}{m(B(x,\sqrt{t}))},$$

where C_1, c_1, C_2, and c_2 are positive constants and $B(x,r)$ is the geodesic ball of radius r centered at the point $x \in M$.

For a signed measure μ in a certain class, let $p^{\mu}(t,x,y)$ be the heat kernel associated with the Schrödinger operator, $\Delta + \mu$. We establish a necessary and sufficient condition on the potential μ for the heat kernel $p^{\mu}(t,x,y)$ also to satisfy the Li-Yau estimate.