Application of the Adams-type inequality to a fully parabolic two-chemical substances chemotaxis system

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This talk is based on a joint work with Prof. Takasi Senba (Fukuoka University).

We will consider the Cauchy problem and the Neumann initial-boundary value problem of the parabolic-parabolic-parabolic chemotaxis system,

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u \nabla v) & \text{in } \Omega \times (0,\infty), \\ v_t = \Delta v - v + w & \text{in } \Omega \times (0,\infty), \\ w_t = \Delta w - w + u & \text{in } \Omega \times (0,\infty), \end{cases}$$

in $\Omega = \mathbb{R}^4$ or open ball $\Omega = B(R) = \{x \in \mathbb{R}^4 \,|\, |x| < R\}$ $(R > 0)$ with nonnegative smooth initial data $(u_0,\, v_0,\, w_0)$.

A Lyapunov functional of the above problems will be constructed and by applying the Adams-type inequality in $H^2(\Omega)$, we will establish global existence and boundedness of solutions to the problem if either

- $\Omega = \mathbb{R}^4$ and $\|u_0\|_{L^1(\Omega)} < (8\pi)^2$,
- $\Omega = B(R)$, $(u_0,\, v_0,\, w_0)$ is radial symmetry and $\|u_0\|_{L^1(\Omega)} < (8\pi)^2$,
- $\Omega = B(R)$ and $\|u_0\|_{L^1(\Omega)} < (8\pi)^2/2$.

The above constants are derived from the critical constant in the Adams-type inequality. Moreover some discussions about the criticality of this condition will be given by observing stationary solutions. This result gives a generalization of [1].

Reference

[1] T. Nagai, T. Senba, K. Yoshida, *Application of the Trudinger-Moser inequality to a parabolic system of chemotaxis.* Funkcial. Ekvac. **40** (1997), 411–433.

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