Stationary solutions of bistable reaction-diffusion equations on some thin tubular domain

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On a thin tubular domain $\Omega \subset \mathbb{R}^N$, we consider the Neumann problem

$$\text{(P)} \quad \begin{cases} \Delta u + f(u) = 0 & x \in \Omega, \\ \partial_\nu u = 0, & x \in \partial\Omega, \end{cases}$$

where $f(u)$ is a bistable nonlinearity. The domain Ω is a tubular neighborhood of a line segment such that the middle part is much thinner than the other part. Since the domain is close to a line segment, the equation (P) is expected to be approximated by some one-dimensional limiting equation. After formally deriving the appropriate limiting equation, we observe that it indeed approximates (P). Then we discuss the solution structure of the limiting equation.