Numerical analysis for structure of stationary solutions for SKT cross-diffusion model

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We are interested in the following strongly-coupled time-dependent system of two competing species with self- and cross-diffusion effect:

$$\begin{cases} u_t = \Delta[(d_1 + \alpha_{11}u + \alpha_{12}v)u] + u(a_1 - b_1u - c_1v) & \text{in } \Omega \times (0,\infty), \\ v_t = \Delta[(d_2 + \alpha_{21}u + \alpha_{22}v)v] + v(a_2 - b_2u - c_2v) & \text{in } \Omega \times (0,\infty), \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0 & \text{on } \partial\Omega, \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x) & \text{in } \Omega, \end{cases}$$

where, $u = u(x,t)$ and $v = v(x,t)$ are non-negative unknown functions, d_i, a_i, b_i, c_i $(i,j = 1,2)$ are all positive constants, α_{ij} $(i,j = 1,2)$ denote non-negative constants, Ω is bounded domain in $\mathbb{R}^N$, $n \geq 1$ with smooth boundary $\partial\Omega$, ν is the outward unit normal vector on $\partial\Omega$. This mathematical model was proposed by Shigesada, Kawasaki and Teramoto in 1979 to investigate segregation phenomena of two competing species with each other in the same habitat area. The effect of cross-diffusion affects the population pressure between two different kinds. It is an interesting problem to see whether this effect may give rise to a spatial segregation or not.

Lou and Ni derived stationary limiting equations of this model to clarify this problem. In this talk, we show numerical results of global structure of stationary solutions for to the stationary limiting equation.