Inverse problems for embedded eigenvalues

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We consider one kind of inverse spectral problems for discrete Schrödinger operators on d-dimensional lattice $\mathbb{Z}^d$. Especially we are interested in the inverse problems concerning embedded eigenvalues in the continuous spectrum and threshold resonances.

Let $n \in \mathbb{N}$ and mutually distinct points $x_1, x_2, \cdots, x_n \in \mathbb{Z}^d$ be fixed. For each $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n) \in \mathbb{R}^n$ we study the Schrödinger operators

$$L = L_\alpha = -\Delta + V_\alpha, \qquad V_\alpha = \sum_{j=1}^{n} \alpha_j \delta_{x_j}$$

on $l^2(\mathbb{Z}^d)$ and it follows that the essential spectrum $\sigma_{ess}(L_\alpha) = [-1, 1]$.

In [IM] it is proved that there is no eigenvalue in $(-1, 1)$ for L_α , so it is only possible to have 1 or -1 as embedded eigenvalues.

Let us consider the set PV:

$$PV = \{\alpha \in \mathbb{R}^n \mid L_\alpha \text{ has an eigenvalue} - 1\}$$

and we call PV the persistent set (variety). Our interest is in structures and geometry of PV and how much information is contained in it about the spectrum of L. Especially we pay attention to the singularities of PV.

reference

[IM] H. Isozaki and H. Morioka, A Rellich type theorem for discrete Schrödinger operators, *Inverse Probl. Imaging* 8 (2014), no. 2, 475-489.