On directional blow-up for a semilinear heat equation with space-dependent reaction

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We consider a nonnegative solution u of the Cauchy problem for a semilinear heat equation with space-dependent reaction $u_t = \Delta u + |x|^\theta u^p$ with the initial data $u_0(x)\,(\not\equiv 0)$ satisfying $\||x|^{\theta/(p-1)}u_0\|_{L^\infty(\mathbf{R}^N)} = M \in (0,\infty)$, where $p > 1$, $0 \le \theta \le (N-2)(p-1)$ and $N \ge 3$. Then, this solution u satisfies that

$$|x|^{\theta/(p-1)}u(x,t) \le v_M(t) \quad \text{ in } \mathbf{R}^N \times (0, T_M),$$

where $v_M(t)$ is a solution of the initial value problem of an ordinary differential equation

$$\begin{cases} v_t = v^p & \text{in } (0,T), \\ v(x,0) = M, \end{cases}$$

and T_M is the blow-up time of v_M, namely,

$$T_M = \frac{M^{-p+1}}{p-1}.$$

We study a weighted solution $|x|^{\theta/(p-1)}u$ which blow up at minimal blow-up time. Such a weighted solution blows up at space infinity in some direction at the time T_M (directional blow-up). We call this direction a *blow-up direction* of the weighted solution $|x|^{\theta/(p-1)}u$. We give a sufficient and necessary condition on u_0 for a weighted solution to blow up at minimal blow-up time. Moreover, we completely characterize blow-up directions of $|x|^{\theta/(p-1)}u$ by the profile of the initial data.

This talk is based on a joint work with Noriaki Umeda (Meiji University).