Rate of convergence to asymptotic profiles for fast diffusion on domains

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Abstract

This talk is concerned with the Cauchy-Dirichlet problem for fast diffusion equations on bounded domains. It is well known that every (possibly sign-changing) weak solution vanishes in finite time at the unique power rate, and therefore, asymptotic profiles for such vanishing solutions are defined as a limit of rescaled solutions, which solve the Cauchy-Dirichlet problem for a fast diffusion equation with a blow-up reaction. Asymptotic profiles are characterized as nontrivial equilibria of the rescaled problem (see pioneer works of Berryman and Holland in 1980s and subsequent results for qualitative results). Recently, Bonforte and Figalli (CPAM, 2021) established an important quantitative result of the convergence of rescaled solutions to positive asymptotic profiles. More precisely, they proved an exponential convergence of nonnegative rescaled solutions to nondegenerate positive asymptotic profiles in a weighted L^2 space with a *sharp rate* (in view of some linearized problems) by developing a *nonlinear entropy method*. In this talk, we shall develop a different approach to prove exponential convergence with rates for nondegenerate asymptotic profiles with definite or changing signs. In particular, if we restrict ourselves to nondegenerate positive asymptotic profiles, we shall exhibit an H_0^1 convergence with the sharp rate. Our method of proofs is based on an energy method rather than entropic one, and a key ingredient is a quantitative gradient inequality established based on an eigenvalue problem with weights, which was already introduced by Bonforte and Figalli and also plays a crucial role in our analysis.