Feynman-Kac methods in the study of non-local Schrödinger operators

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Classical Schrödinger operators of the form $H = -\frac{1}{2}\Delta + V$, where Δ is the Laplacian and V is a multiplication operator called potential, are central objects in mathematical quantum theory, describing the Hamiltonian of a particle, and they have been studied for many years. An eigenfunction at the lowest-lying eigenvalue, that is, a function $\varphi_0 \in \mathrm{Dom}(H) \setminus \{0\} \subset L^2(\mathbb{R}^d)$ satisfying $H\varphi_0 = \lambda_0\varphi_0$ with $\lambda_0 = \inf \mathrm{Spec}(H)$ is called a *ground state* of H, whenever it exists.

The operator $H = (-\Delta + m^2)^{1/2} - m + V$, $m \geq 0$, describes the Hamiltonian of a so-called semi-relativistic quantum particle of rest mass m, which goes beyond the class of classical Schrödinger operators. In this case the Laplacian, which is a differential operator, is replaced by a pseudo-differential operator. The relativistic Schrödinger operator can be seen as a particular case of a family of operators $H = \Psi(-\Delta) + V$, where Ψ is a specific family of functions (to be discussed in the talk), which we call *non-local Schrödinger operators.*

In this talk my main aim is to explain a useful relationship between ground states of non-local Schrödinger operators and a class of random processes, called *Feynman-Kac-type formula.* I will also illustrate how these stochastic representations can be used to study the properties of ground states.