Stability of non-zero equilibrium states for the viscous Burgers equation with a delay effect

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In this talk, we consider the viscous Burgers equation with a time delay

$$\partial_t \rho - \nu \partial_x^2 \rho + \partial_x \left(\rho V(\rho_\tau) \right) = 0 \tag{1}$$

in the one-dimensional whole space. Here $\rho = \rho(t, x)$ is an unknown function, ν is a positive constant, and $V = V(\rho)$ denotes a given smooth function that depends on ρ. Also, we use a notation that $\rho_\tau(t, x) = \rho(t - \tau, x)$ with a positive constant τ called delay parameter.

Recently, Kubo and Ueda (2022) treated the Cauchy problem of (1) and obtained the nonlinear stability of the zero solutions. Furthermore, they derived the condition for the initial history and the time delay to get the nonlinear stability. We remark that their result does not assign the smallness for both the initial history and the time delay. On the other hand, there is no result for the stability of the non-zero equilibrium state for (1), and our main purpose of this talk is to investigate the condition to get the linear stability of the non-zero equilibrium state.

The linear stability is analyzed by using the characteristic equation of the corresponding eigenvalue problem. If our equation does not have a delay effect, the characteristic equation is given by a polynomial equation. On the other hand, if our equation has a delay effect, the characteristic equation becomes a transcendental equation, and it is difficult to analyze it. In this situation, we apply the useful known result concerned with the characteristic equation for the ordinary delay differential equations and try to get the sharp stability condition for the viscous Burgers equation with delay.

Furthermore, if there is enough time, we would like to expand the discussion to nonlinear stability.