Blow up of solutions of semilinear wave equations with time-dependent propagation speed and damping

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In this talk we consider the Cauchy problem

$$\begin{cases} u_{tt} - a(t)^2 \Delta u + b(t) u_t = |u|^p \,, & t > 0,\ x \in \mathbf{R}^n, \\ \\ u(0,x) = \varepsilon u_0(x),\ u_t(0,x) = \varepsilon u_1(x), & x \in \mathbf{R}^n, \end{cases}$$

where $a \in C^1([0,\infty)),\ b \in C([0,\infty))$, $\Delta = \partial_1^2 + \cdots + \partial_n^2,\ \partial_j = \partial/\partial x^j,\ j = 1, \cdots, n,$ $(x^1, \cdots, x^n) \in \mathbf{R}^n$, $p > 1$, and $\varepsilon > 0$ is a small parameter.

This equation is a generalized version of the wave equations in the Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime and the de Sitter spacetime. The FLRW and de Sitter spacetimes are known as standard model of cosmology. By dividing the damping type into effective and scattering cases, we show, in terms of the growth of the support of the solution, that blow-up in a finite time occurs for the equation as well as upper bounds of the lifespan of blow-up solutions.

This talk is based on a joint work with Y. Wakasugi (Hiroshima University).

References

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