Asymptotic behavior of bifurcation curves and related topics

Tetsutaro Shibata

Graduate School of Advanced Science and Engineering,

Hiroshima University, Higashi-Hiroshima, 739-8527, Japan

We consider

$$
\begin{aligned}
-u''(t) &= \lambda\left(u(t)+g(u(t))\right), \quad t \in I := (-1,1), && (0.1)\\
u(t) &> 0, \quad t \in I, && (0.2)\\
u(-1) &= u(1) = 0, && (0.3)
\end{aligned}
$$

where $\lambda > 0$ is a parameter, and in what follows, we assume that $g(u)$ satisfies the following conditions.

(A.1) $g(u) \in C^1(\mathbb{R})$ and $u + g(u) > 0$ for $u > 0$.

(A.2) $g(u + 2\pi) = g(u)$ for $u \in \mathbb{R}$.

It is well known (cf. [T. Laetsch, 1970]) that if $u + g(u) > 0$ for $u > 0$, then by time-map method, we find that λ is parameterized by using $\alpha = \|u\|_\infty$, such as $\lambda = \lambda(\alpha)$ and is a continuous function of $\alpha > 0$. Since λ depends on g, we sometimes write $\lambda = \lambda(g, \alpha)$. The study of the structures of the bifurcation curves is one of the main topics in bifurcation analysis, and there are quite many works concerning the properties of bifurcation diagrams. In particular, the qualitative properties of the oscillatory bifurcation diagrams have been studied intensively. In this talk, we focus on the study whether $\lambda(g, \alpha)$ inherits the oscillatory properties of $g(u)$ or not if $g(u)$ is a periodic function.

E-mail: tshibata@hiroshima-u.ac.jp