Global weak solvability of an N-dimensional initial-boundary value problem for the moisture transport model for porous materials*

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In this talk we consider the following initial-boundary value problem:

$$\begin{cases} \dfrac{\partial}{\partial t}\psi(u) = \nabla \cdot \big(\lambda(u)\nabla(u+g)\big), & t \in (0,T),\ x \in \Omega, \\ \dfrac{\partial}{\partial n}(u+g) = 0, & t \in (0,T),\ x \in \partial\Omega, \\ u(0,x) = u_0(x), & x \in \Omega, \end{cases} \tag{P}$$

where $T > 0$ and $\Omega \subset \mathbb{R}^N$ $(N \in \mathbb{N})$ is a bounded domain with smooth boundary $\partial\Omega$, and where $\psi, \lambda \in C^1(\mathbb{R})$, $g\colon (0,T)\times\Omega \to \mathbb{R}$ and $u_0\colon \Omega \to \mathbb{R}$ are given functions, and $u\colon (0,T)\times\Omega \to \mathbb{R}$ is an unknown function, $\frac{\partial}{\partial n}$ denotes the outward normal derivative on $\partial\Omega$. The model (P) describes the moisture transport in porous materials. The function u represents the water chemical potential. Also, the functions ψ and λ mean the water content and the water conductivity by the water pressure gradient, respectively. In the previous work [1] global existence and uniqueness of weak solutions to (P) were obtained when $\Omega = (0,1)$ under the restriction

$$\frac{\lambda(r)}{\psi'(r)} \equiv \alpha \quad (r \in \mathbb{R})$$

for some constant $\alpha > 0$. The purpose of this talk is to relax the restriction and to establish global weak solvability of the above N-dimensional initial-boundary value problem.

Theorem 1. *Assume $g \in L^2(0,T;H^1(\Omega))$ and*

$$0 < \alpha \le \frac{\lambda(r)}{\psi'(r)} \le \beta, \quad 0 < \lambda(r) \le K, \quad \left|\frac{\lambda'(r)}{\psi'(r)}\right| \le L \quad (r \in \mathbb{R})$$

with some constants $\alpha, \beta, K, L > 0$. Let $u_0 : \Omega \to \mathbb{R}$ with $\psi(u_0) \in L^2(\Omega)$. Then there exists a weak solution $u : (0,T)\times\Omega \to \mathbb{R}$ of (P) *such that $\psi(u) \in L^2(0,T;H^1(\Omega))$ and*

$$\int_0^T\!\!\int_\Omega \left(-\psi(u)\frac{\partial\varphi}{\partial t} + \left(\frac{\lambda(u)}{\psi'(u)}\nabla\psi(u) + \lambda(u)\nabla g\right)\cdot\nabla\varphi\right)dxdt = \int_\Omega \psi(u_0(\cdot))\varphi(0,\cdot)\,dx$$

for all $\varphi \in C_c^\infty([0,T);H^1(\Omega))$.

References

[1] Y. Chiyo, H. Terasaki, Y. Tsuzuki and T. Yokota. Global existence and uniqueness of weak solutions to a one-dimensional moisture transport model for porous materials, *Evol. Equ. Control Theory* 14(6):1614–1637, 2025.

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