

Continuum limit of resonances for discrete Schrödinger operators

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We consider resonances for discrete and continuous Schrödinger operators. We show that the resonances of discrete Schrödinger operators converge to resonances of continuous Schrödinger operators in the continuum limit. The potential is supposed to be a sum of an exterior dilation analytic function and an exponentially decaying function. We may treat singular potentials such as the Coulomb potential by approximating them by a mollifier in the continuum limit. In the proof, we treat resonances by the complex distortion method in the Fourier space and we follow the strategy by Nakamura and Tadano [5] in the discussion of the continuum limit.

In quantum mechanics, resonances are complex numbers which correspond to quasi-steady states and they are closely related to the scattering theory (see, for example, Cycon, Froese, Kirsch and Simon [1] and Dyatlov and Zworski [2]).

Continuum limit for discrete Schrödinger operators has been studied recently by several authors. Nakamura and Tadano [5] proved the convergence of discrete eigenvalues when the potential is continuous. Isozaki and Jensen [3] discussed the scattering theory in the continuum limit.

This talk is based on a joint work [4] with Shu Nakamura.

References

- [1] H. L. Cycon, R. G. Froese, W. Kirsch, B. Simon: *Schrödinger operators with application to quantum mechanics and global geometry*, Springer-Verlag, 1987.
- [2] S. Dyatlov, M. Zworski: *Mathematical Theory of Scattering Resonances*, GSM 200, American Math. Soc., 2019.
- [3] H. Isozaki, A. Jensen: Continuum limit for lattice Schrödinger operators, *Rev. Math. Phys.* **34** (2022), 2250001.
- [4] K. Kameoka, S. Nakamura: Continuum limit of resonances for discrete Schrödinger operators, *J. Math. Phys.* **66** 072105 (2025), 1-13.
- [5] S. Nakamura, Y. Tadano: On a continuum limit of discrete Schrödinger operators on square lattices, *J. Spectr. Theory* **11** (2021), 355–367.