

Mathematical analysis of conductivity equation with imperfect bonding interface

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Abstract

This talk focuses on recent developments in the mathematical analysis of systems with imperfect bonding interfaces.

There are two types of imperfect bonding interfaces: low conductivity-type (LC-type) interfaces, which approach insulating behavior in the membrane limit, and high conductivity-type (HC-type) interfaces, which approach highly conducting behavior. Conceptually, these imperfect bonding conditions bridge the extreme regimes: the ideal interface and an insulator (for the LC-type) or a perfect conductor (for the HC-type) as the interface parameter varies (known as interface or Kapitza resistance in the LC-type case). A series of studies on field concentration problems by Dong et al. [1–3] supports this intuition. In the small-resistance regime for the LC-type interface, we show that as the interface resistance approaches zero, the solution to the governing partial differential equation converges to that of the ideal interface [5]. A neutral inclusion occurs when the effects of the core and the thin shell are critically balanced so that the background field remains completely unperturbed. We present a result on geometric rigidity characterized by neutrality under all uniform fields [4].

This talk is based on joint work [4–6].

References

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