Modular functions and Quadratic forms
- Number theoretic delights -

December 21st

Masanobu Kaneko (Kyushu Univ.)
Title: On the elliptic modular function $j(\tau)$
Abstract: In the spirit of the theme of the conference, we review several works on the modular function $j(\tau)$, both classic and rather recent. The nature or the behavior of $j(\tau)$ at elliptic, parabolic, and hyperbolic points of the (semi) upper half-plane is of our main concern.

Ken Ono (Emory Univ.)
Title: Special numbers and special functions: Part 1
Abstract: This lecture will be about two topics in the theory of mock modular forms.
I. Quantum Modular forms and Ramanujan's Radial Limits (joint work with Amanda Folsom and Rob Rhoades)
II. Weierstrass mock modular forms and the arithmetic of elliptic curves (joint work with Claudia Alfes, Michael Griffin, and Larry Rolen).
This lecture will discuss recent results

Kenichi Bannai (Keio Univ.)
Title: On the p-adic analogue of Eisenstein-Kronecker series
Abstract: The Eisenstein-Kronecker series is an elliptic analogue of the classical polylogarithm function. In this talk, we overview the results concerning the construction of the p-adic analogue of the Eisenstein-Kronecker series for elliptic curves with complex multiplication. We then give some results concerning the relation between these functions and the special values of p-adic L-functions of Hecke characters associated to the field of complex multiplication.

Fumiya Amano (Kyushu Univ.)
Title: Arithmetic Milnor invariants and modular forms
Abstract: This is a summary of my recent work. I will talk about arithmetic of Rédei triple symbols and 4-th multiple generalization, and then a relation between Redei symbols and modular forms.

December 22nd

Takeshi Ogasawara (Oyama National College of Technology)
Title: On Hecke modules generated by eta-quotients of weight one
Abstract: Eta-quotients of weight one are often related to the theta series associated to ideal class characters of quadratic fields. In this talk, we will consider some Hecke modules generated by eta-quotients of weight one, and observe a relationship between them and ideal class groups of corresponding imaginary quadratic fields.

Ken Ono (Emory Univ.)
Title: Special numbers and special functions: Part 2
Abstract: (see the abstract of the first lecture.)

Fumihiro Sato (Rikkyo Univ.)
Title: Zeta functions of quadratic mappings
Abstract: Let (G, ρ, V) be an irreducible regular prehomogeneous vector space defined over $\mathbb{Q}$ and (G, ρ^*, V^*) be its dual. Denote by $P(v)$ and $P^*(v^*)$ the irreducible relative invariants of (G, ρ, V) and (G, ρ^*, V^*), respectively. Let W be a finite dimensional $\mathbb{C}$-vector space with $\mathbb{Q}$-structure and W^* its dual and suppose that there exist quadratic mappings $Q : W \to V$ and $Q^* : W^* \to V^*$ satisfying certain compatibility conditions (non-degenerate dual quadratic mappings). Then we expect that the polynomials $\tilde{P}(w) = P(Q(w)$ and $\tilde{P}^*(w^*) = P^*(Q^*(w)^*$ have interesting arithmetic properties. For example, $\tilde{P}$ and $\tilde{P}^*$ satisfy local functional equation over $\mathbb{R}$ similar to the local functional equations satisfied by relative invariants of regular prehoogeneous vector spaces. In this talk we explain how to define global zeta functions for non-degenerate dual quadratic mappings over prehomogeneous vector spaces. Non-trivial examples of such quadratic mappings can be obtained from representations of Clifford algebras, which include the quadratic mappings inducing the Hopf fibrations $S^3 \to S^2$, $S^7 \to S^4$ and $S^{15} \to S^8$ as special cases.

Takashi Ono (Johns Hopkins Univ.)
Title: Quest for "Golden Numbers"
Abstract: By the (classical) Golden Number we mean a real number $\lambda = 1.61803\cdots$ which is the unique positive real root of the quadratic equation $X^2 = X + 1$. Another root $\mu = 1 - \lambda$ is negative. We have the splitting $X^2 - X - 1 = (X - \lambda)(X - \mu)$ which provides with the diagonalization of the 2x2-matrix $A = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 1 \end{smallmatrix}\right) \sim B = \left(\begin{smallmatrix} \lambda & 0 \\ 0 & \mu \end{smallmatrix}\right)$. Since A generates the (classical) Fibonacci sequence in an obvious way, we obtain the explicit formula and representation of the (classical) Golden number as the limit of ratios of (classical) Fibonacci numbers. We try to generalize all these for certain family of real polynomials and get lots of Golden Numbers and/or Silver ones...

December 23rd

Kazuhiro Hikami (Kyushu Univ.)
Title: Quantum modular form and quantum invariant
Abstract: We will review on a relationship between quantum modular form and quantum invariant of knots and 3-manifolds.

Kiichiro Hashimoto (Waseda Univ.)
Title: Missing link between vanishing of quaternion theta series and nonvanishing cycles on modular curves
Abstract: Classical basis problem for modular forms is to ask if one can find a basis of the space of modular forms with prescribed level and weight,from the theta series associated with definite quaternion algebras over **Q**. The answer was given to a complete extent by Eichler, Hijikata, Pizer, Shemanske, etc. It can be viewed as one of the explicit version of Eichler-Jacquet-Langlands correspondence. There are several refined versions of the same problem restricting the type of theta series or modular forms. An interesting example arise from the observation that the type number (i.e., the number of isomorphic classes of maximal orders) of the definite quaternion algebra over **Q** with discriminant q coincides with the dimension of the space of weight 2 new forms of level q, on which the Atkin-Lehner involutions for each prime divisor of q acts as -1. Such relation has been extended to the Eichler orders or more general orders. This leads us to study the theta series of orders and to ask the refined basis

problem. In this talk we shall discuss the significance of this problem. When the level of orders $\mathcal{O}$ is square free, it was known by Gross, and Böcherer, Schulze-Pillot that a linear relation of certain ternary theta series attached to such orders corresponds, throgh the Shimura lifting, to a primitive cusp form of weight 2 having root number +1, whose L-functions vanish at $s = 1$. Based on numerical computations we propose a new conjecture by which the vanishing of the L-functions at $s = 1$ for $f \in S_2^0(M)$ (M arbitrary), should be controlled by the linear relations of (quaternary) theta series of various levels qN, including the case when $L(f, s)$ has root number -1. This suggests a possibility (= my dream) of finding 0-cycles on modular curves which give rational points of infinite order on their jacobian varieties.

Nobushige Kurokawa (Tokyo Institute of Technology)
Title: Multiple sine functions and Eisenstein series
Abstract: I report an application of multiple sine functions to limit values of Eisenstein series of non-classical type. For this purpose I explain the theory multiple sine functions and multiple cotangent functions. I remark on relations to absolute zeta functions also.

Eiichi Bannai (Shanghai Jiao Tong Univ.)
Title: "Gauss sums and Legendre polynomials" in algebraic combinatorics
Abstract: Professor Takashi Ono published a series of articles entitled "Gauss sums and Legendre polynomials" in Sugaku Seminar (November 1986–April 1987). The main point of the series was to point out an analogy between the theory of harmonic analysis on finite abelian groups and harmonic analysis on the sphere in the Euclidean space. I was very much fascinated with the series, because that is exactly what we are trying to pursue in algebraic combinatorics. In algebraic combinatorics, we study codes and designs on the sphere (using harmonic analysis on the sphere) and also codes and designs on various association schemes (using harmonic analysis on association schemes). We will review this situation, including our recent studies on Euclidean designs (which is a generalization of spherical designs) and on relative designs (which is a generalization of designs) on association schemes. We will emphasize the analogy and also some differences between these two theories.