On Backward Mean Curvature Flow for Equifocal Submanifolds in Symmetric Spaces of Compact Type

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Introduction

M: a manifold, N: a Riemannian manifold

$\{f_t\}_{t\in[0,T)}$: a one-parameter C^∞-family of immersions of M into N with mean curvature v.f. $\boldsymbol{H}_t$, $T \in \mathbb{R}_+ \cup \{\infty\}$

$\{f_t\}_t$: mean curvature flow (MCF)

$$\iff F_*(\frac{\partial}{\partial t}) = \boldsymbol{H}_t \text{ (where } F(\cdot, t) = f_t(\cdot))$$

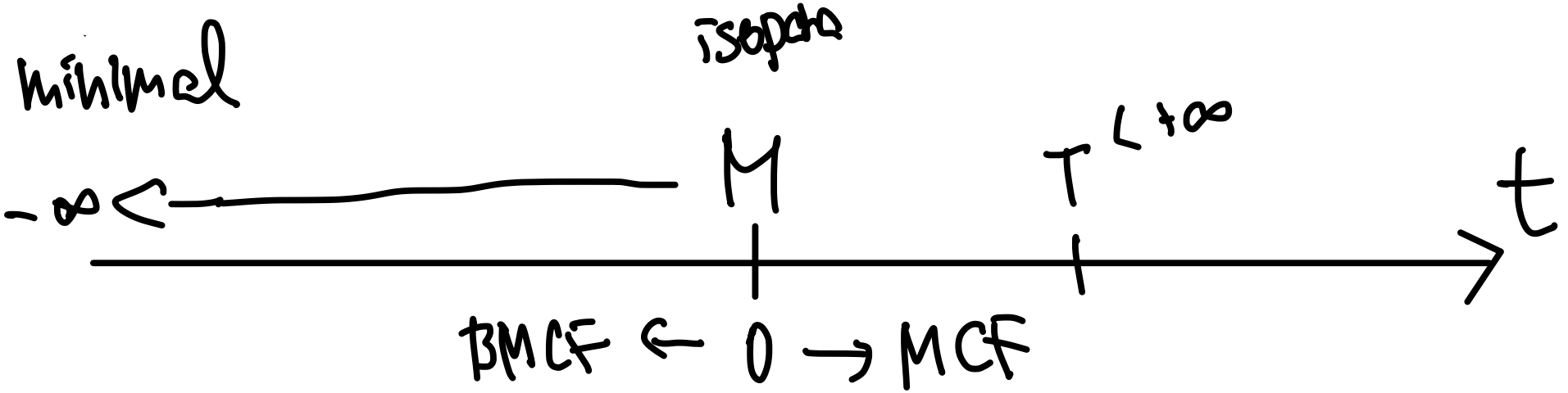
$\{f_t\}_t$: backwad MCF (BMCF)

$$\iff F_*(\frac{\partial}{\partial t}) = -\boldsymbol{H}_t$$

$\{M_t = f_t(M)\}_t$: MCF/BMCF if $\{f_t\}_t$: embeddings and MCF/BMCF

Introduction

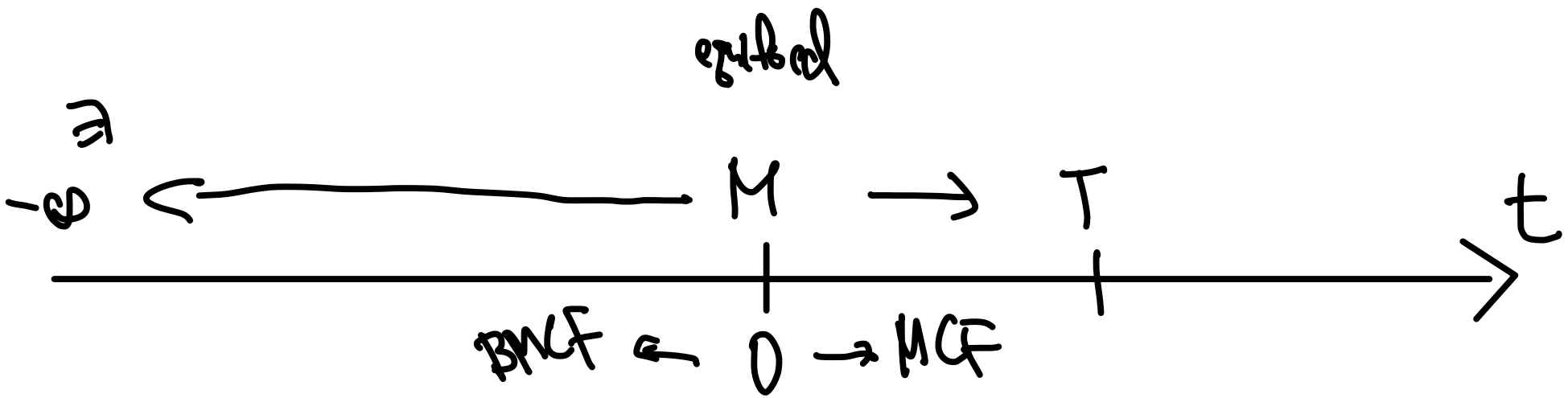
- Liu-Terng (2009): the MCF starting from isoparametric submanifolds (and their focal submanifolds) in the Euclidean sp. $\mathbb{R}^N$ and the sphere S^N
- Liu-Terng (2020): ancient solutions of MCF (= long-time solutions of BMCF) starting from isopara. submfds in $\mathbb{R}^N$ and S^N



Introduction

• Koike (2011): the MCF starting from equifocal submanifolds (and their focal submanifolds) in symmetric sp. of compact type

• Liu-Radeschi (2022): the MCF starting from leaves of polar foliation on symmetric space of non-negative



Introduction

Ancient solutions play an important role in the study of MCF/BMCF; e.g. singularities of MCF, rigidity of BMCF

Theorem (Huisken-Sinestrari)

MCF $\{M_t^n\}_t$ *in* S^{n+1} *is either a shrinking sphereical cap or a stationary solution if one of the followings holds for all* $t < 0$*:*

(1) $\exists B < 4n$*: constant s.t.* $|A_t|^2 < e^{-Bt}|H_t|^2$

(2) $n \geq 3$ *and*

$$|A_t|^2 - \frac{1}{n-1}|H_t|^2 \leq 2$$

Introduction

• Furthermore, in the study of rigidity, Liu-Terng (2020) derived some estimates ($\frac{|A_t|^2}{|H_t|^2}$ etc.) for BMCF starting from isoparametric hypersurfaces in the sphere

• Motivated above studies, we investigate some estimates ($\frac{|A_t|^2}{|H_t|^2}$ etc.) for BMCF starting from equifocal hypersurfaces in symmetric sp. of compact type

Contents of this talk

1. Introduction
2. Equifocal submanifolds and infinite-dimensional isoparametric submanifolds
3. MCF/BMCF for equifocal submanifolds
4. MCF/BMCF and Hermann actions

Equifocal submfds and infinite-dim. isoparametric submfds

G/K: symmetric sp. of cpt type

M: a connected, immersed compact submanifold of G/K

$\mathcal{F}(M)$: the tangential focal set $= \{$all critical pts. of $\exp^{\perp} : T^{\perp}M \to N\}$

Definition (Terng-Thobergsson)

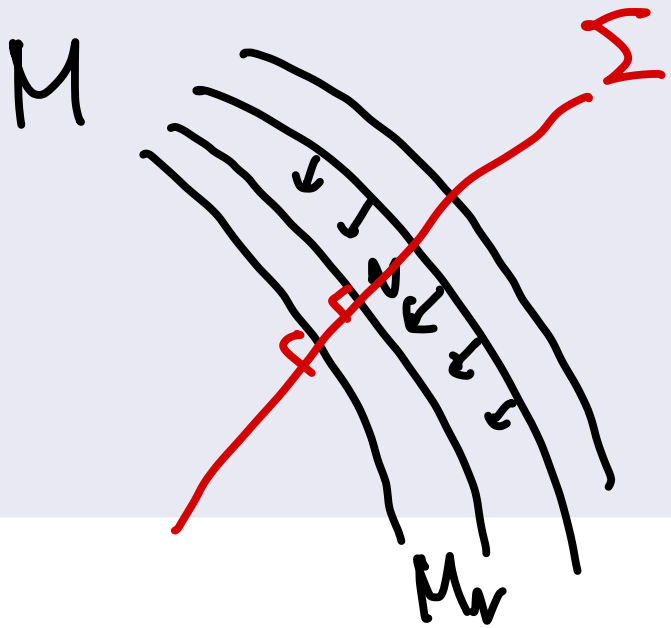
M: equifocal if it satisfies the following conditions:

(1) M has a flat section Σ.

(2) $\mathrm{Hol}^{\perp}(M)$ is trivial.

(3) $\mathcal{F}(M)$ is invariant under

the parallel transformation w.r.t. $\nabla^{\perp}$



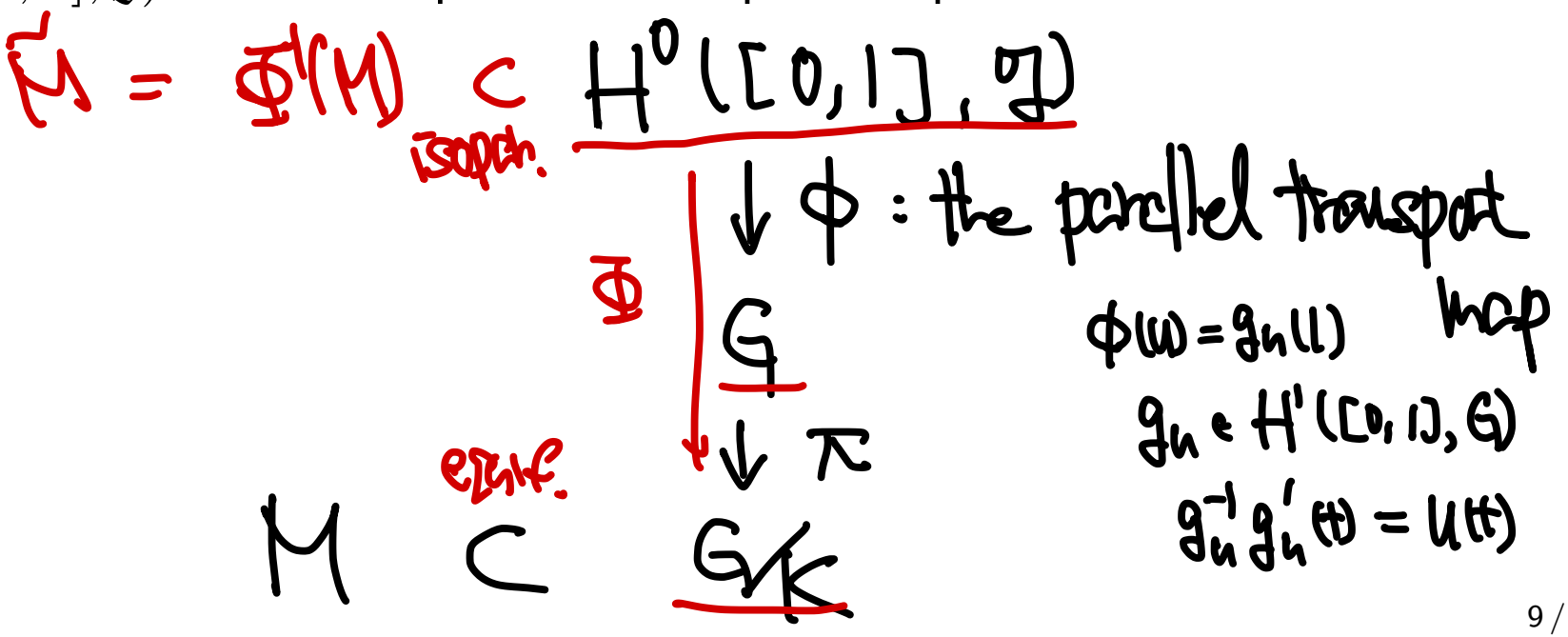
Equifocal submfds and infinite-dim. isoparametric submfds

In order to study equifocal submanifolds, we make use of the following Riemannian submersion due to Terng-Thobergsson:

G/K: symmetric sp. of cpt type, $\pi : G \to G/K$: the natural proj.

$H^0([0,1], \mathfrak{g})$: the Hilbert space which consists of all H^0-curves in $\mathfrak{g}$

$\phi : H^0([0,1], \mathfrak{g}) \to G$: the parallel transport map for G



Equifocal submfds and infinite-dim. isoparametric submfds

Here, we recall the notion of infinite dim. isoparametric submfds.

V: a (separable) Hilbert space

Definition (Palais-Terng)

$\widetilde{M} \subset V$: proper Fredholm (PF) if it satisfies the followings:

(1) The normal exponential map $\widetilde{\exp}^{\perp}$ is a Fredholm map.

(2) The restriction of $\widetilde{\exp}^{\perp}$ to the ball normal bdl. of any radius is proper.

Fact. For a PF submfd. $\widetilde{M}$, the shape operators of $\widetilde{M}$ are self-adjoint and compact operators.

Equifocal submfds and infinite-dim. isoparametric submfds

$\widetilde{M} \subset V$: a PF submanifold of V

Definition (Terng)

$\widetilde{M}$: isoparametric if it satisfies

(1) $\mathrm{Hol}^{\perp}(\widetilde{M})$ is trivial.

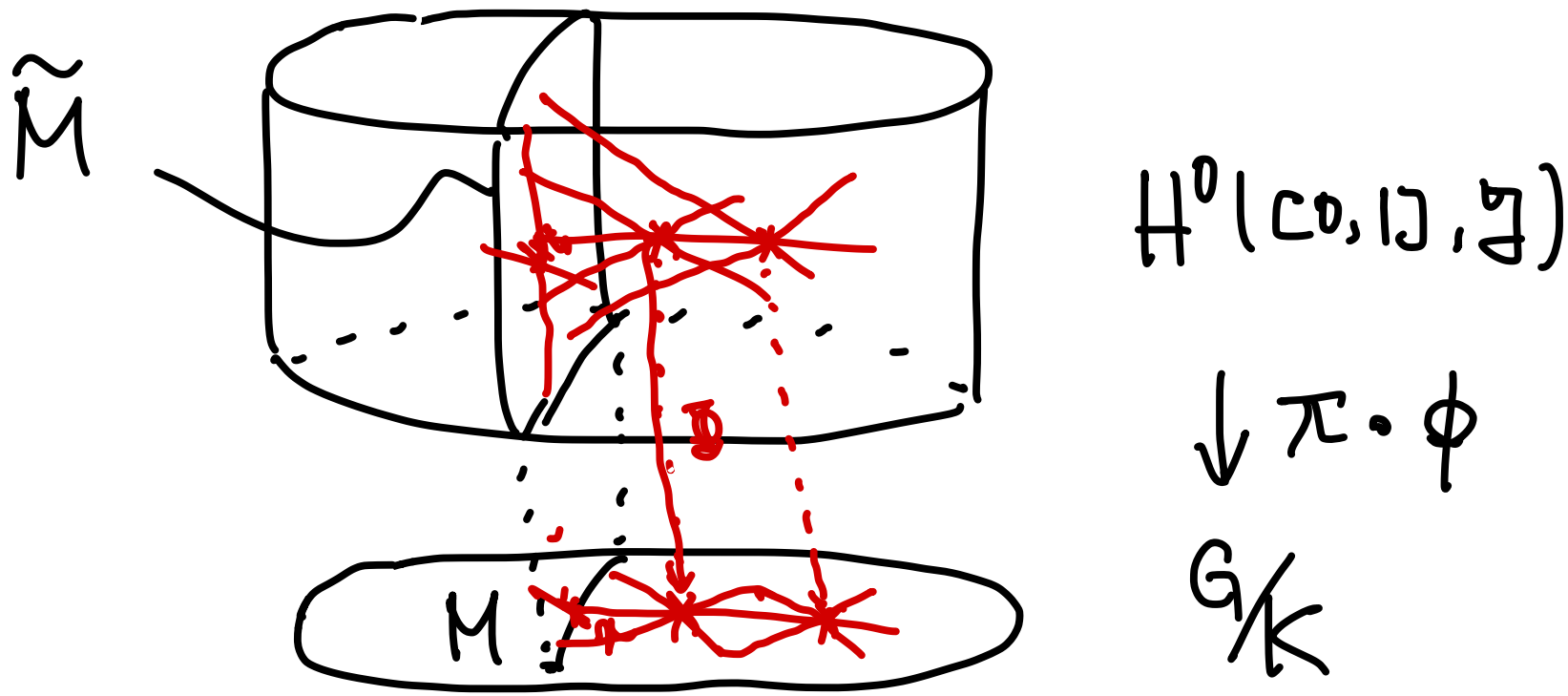
(2) For any parallel normal v.f. $\tilde{v}$ of $\widetilde{M}$, the set of all eigenvalues of the shape operator $\widetilde{A}_{\tilde{v}_u}$ is independent of the choice of the point $u \in \widetilde{M}$ with considering the multiplicities.

Equifocal submfds and infinite-dim. isoparametric submfds

Fact. M: an equifocal submanifold of G/K

(1) $\widetilde{M} := (\pi \circ \phi)^{-1}(M)$ is a isoparametric submfd of $H^0([0,1], \mathfrak{g})$.

(2) For $u \in \widetilde{M}$ and $x = (\pi \circ \phi)(u)$, the following relation holds under the identification of $T^\perp_x M$ and $T^\perp_u \widetilde{M}$: $\mathcal{F}(M, x) = \mathcal{F}(\widetilde{M}, u)$



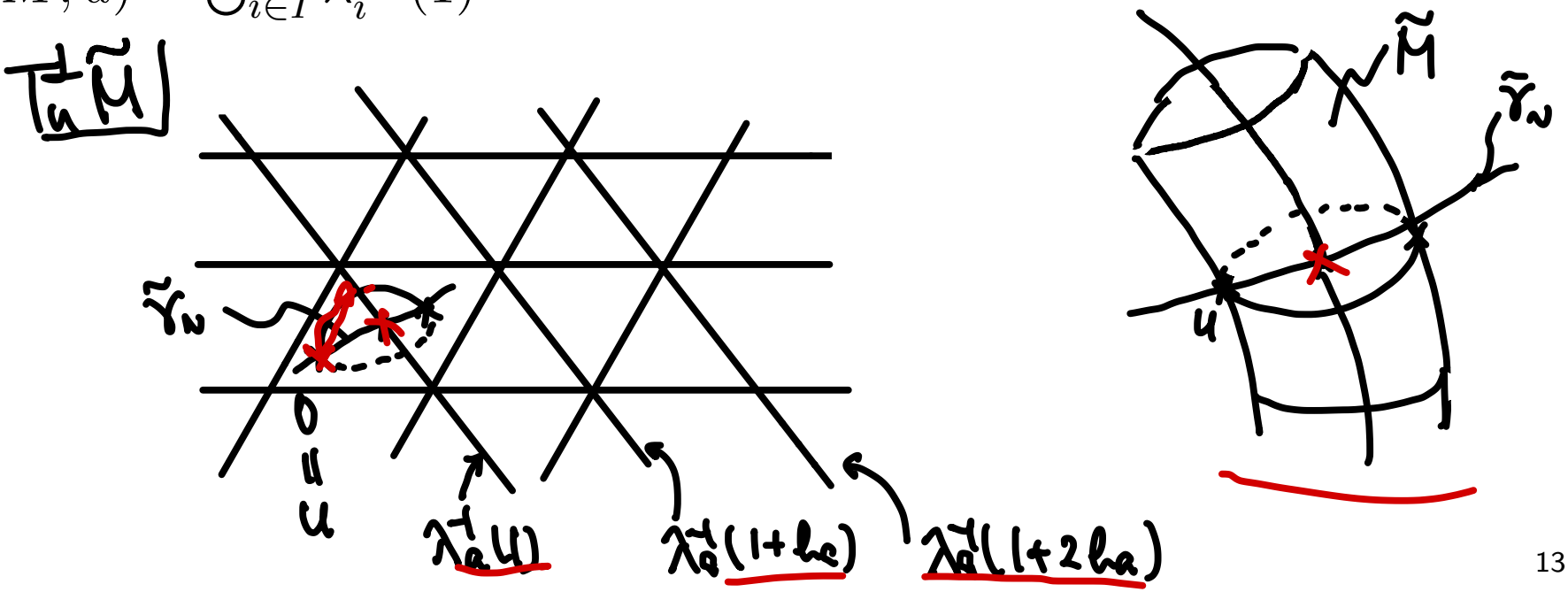
Equifocal submfds and infinite-dim. isoparametric submfds

Since $\{\widetilde{A}_v \mid v \in T^\perp_u \widetilde{M}\}$ is a commutative family, we have:

$$T_u\widetilde{M} = \overline{(E_0)_u \oplus \bigoplus (E_i)_u} \quad (E_i := \bigcap_{v \in T^\perp_u \widetilde{M}} \mathrm{Ker}(\widetilde{A}_v - (\lambda_i)_u(v)\mathrm{id}))$$

ℓ^x_i: the affine hyperplane $(\lambda_i)^{-1}_u(1)$ in $T^\perp_u \widetilde{M}$

$\mathcal{F}(\widetilde{M}, u) = \bigcup_{i \in I} \lambda_i^{-1}(1)$



Equifocal submfds and infinite-dim. isoparametric submfds

$\mathcal{F}(\widetilde{M}, u) = \bigcup_{a=1}^{\bar{r}} \{\lambda_i^{-1}(1 + b_a j) \mid j \in \mathbb{Z}\}$

$\mathcal{PC}(\widetilde{M}, u) =$ the set of all principal curvatures of $\widetilde{M}$ at u

$= \bigcup_{a=1}^{\bar{r}} \{\lambda_{a,j} \mid j \in \mathbb{Z}\}$ $(\lambda_{a,j} := \dfrac{\lambda_a}{1 + b_a j})$

It is shown that, for each $a \in \{1, \ldots, \bar{r}\}$,

the multiplicities of $\lambda_{a,2j}$ $(j \in \mathbb{Z})$ are constant $(=: m_a^e)$

the multiplicities of $\lambda_{a,2j+1}$ $(j \in \mathbb{Z})$ are constant $(=: m_a^o)$

Then, $\{(\lambda_a, b_a, m_a^e, m_a^o) \mid a = 1, \ldots, \bar{r}\}$ is called the tangential focal data of M.

Equifocal submfds and infinite-dim. isoparametric submfds

Heintze-Liu-Olmos proved:

Fact. M: equifocal, $\widetilde{M} := (\pi \circ \phi)^{-1}(M)$

Then, $\widetilde{M}$ is a regularized isoparametric submaifold. Furthermore, $\widetilde{M}$ admits the regularized mean curavature v.f. $\widetilde{\boldsymbol{H}}$ and $\widetilde{\boldsymbol{H}}$ is the horizontal lift of the mean curvature v.f. of M to $H^0([0,1], \mathfrak{g})$.

Here, for a PF submanifold $\widetilde{M}$,

$\widetilde{M}$: regularizable if $\mathrm{Tr}\widetilde{A}_v^2 < \infty$ and $\mathrm{Tr}_r \widetilde{A}_v < \infty$ for all $v \in T_u^\perp \widetilde{M}$ and $u \in \widetilde{M}$.

MCF/BMCF for equifocal submanifolds

- In order to study MCF/BMCF for equifocal submanifolds, we lift up MCF/BMCF by using the Riem. sub. $\pi \circ \phi : H^0([0,1], \mathfrak{g}) \to G/K$.
- For this, we recall the notion of regularized MCF/BMCF.

V: a Hilbert space, $\widetilde{M}$: a Hilbert manifold

$\{\widetilde{f}_t\}_t$: a 1-parameter C^∞-family of embeddings of $\widetilde{M}$ of V, $\widetilde{M}_t := \widetilde{f}_t(\widetilde{M})$

$\{\widetilde{M}_t\}_t$: regularized mean curvature flow

$\iff$ $\widetilde{M}_t$: regularized submfd and $\widetilde{F}_*(\dfrac{\partial}{\partial t}) = \widetilde{\boldsymbol{H}}_t$ $(\widetilde{F}(\cdot, t) = \widetilde{f}_t(\cdot))$

$\{\widetilde{M}_t\}_t$: regularized backward mean curvature flow

$\iff$ $\widetilde{M}_t$: regularized submfd and $\widetilde{F}_*(\dfrac{\partial}{\partial t}) = -\widetilde{\boldsymbol{H}}_t$

MCF/BMCF for equifocal submanifolds

M: an equifocal submfd of G/K

M_w: the parallel submfd of M along $w \in T^{\perp}_x M$ $(x \in M)$

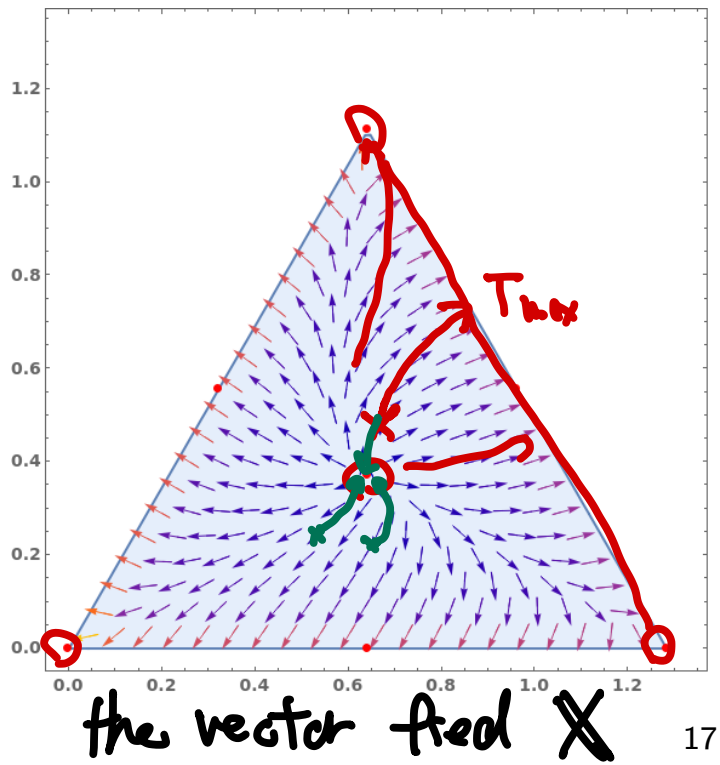
$\widetilde{M}_w := (\pi \circ \phi)^{-1}(M_w)$: regularized isoparametric submfd of $H^0([0,1], \mathfrak{g})$

$\widetilde{\boldsymbol{H}}^w$: the regularized mean curvature v.f. of $\widetilde{M}_w$

$u \in (\pi \circ \phi)^{-1}(x)$

$\boldsymbol{X}$: a vector fied on the fundamental domain $\widetilde{C} \ni 0$ defined by

$$\boldsymbol{X}_w = (\widetilde{\boldsymbol{H}}^w)_{u+w}, \quad w \in \widetilde{C}$$



MCF/BMCF for equifocal submanifolds

$\xi^w : (-T_{\min}, T_{\max}) \to \widetilde{C}$: the maxi. integral curve of $\boldsymbol{X}$ with $\xi^w(0) = w$

Lemma (cf. Koike (2011))

(1) $\{\widetilde{M}_{\xi^w(t)}\}_{t\in[0,T_{\max})}$ *is the regularized MCF starting from* $\widetilde{M}_w$.

$\{\widetilde{M}_{\xi^w(-t)}\}_{t\in[0,T_{\min})}$ *is the regularized BMCF starting from* $\widetilde{M}_w$.

(2) $\{M_{\xi^w(t)}\}_{t\in[0,T_{\max})}$ *is the MCF starting from* M_w.

$\{M_{\xi^w(-t)}\}_{t\in[0,T_{\min})}$ *is the BMCF starting from* M_w.

MCF/BMCF for equifocal submanifolds

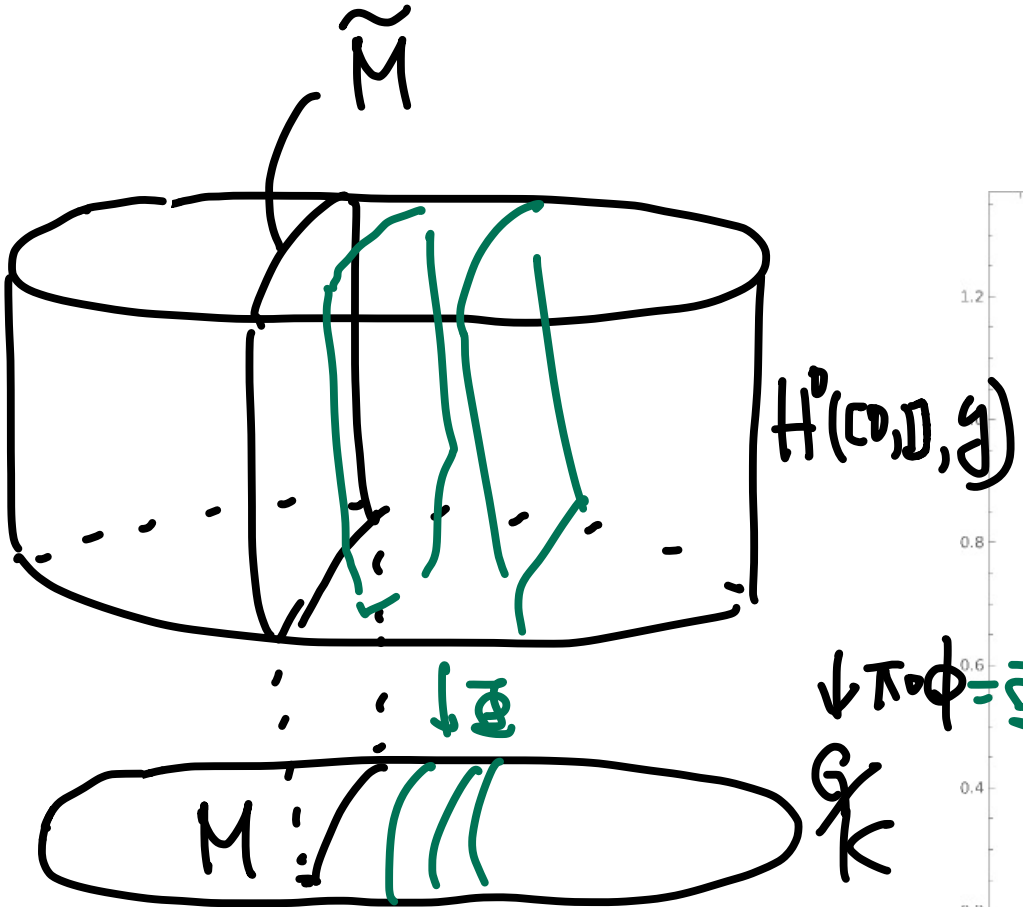
Fact (Koike (2011), Liu-Radeschi (2020)).

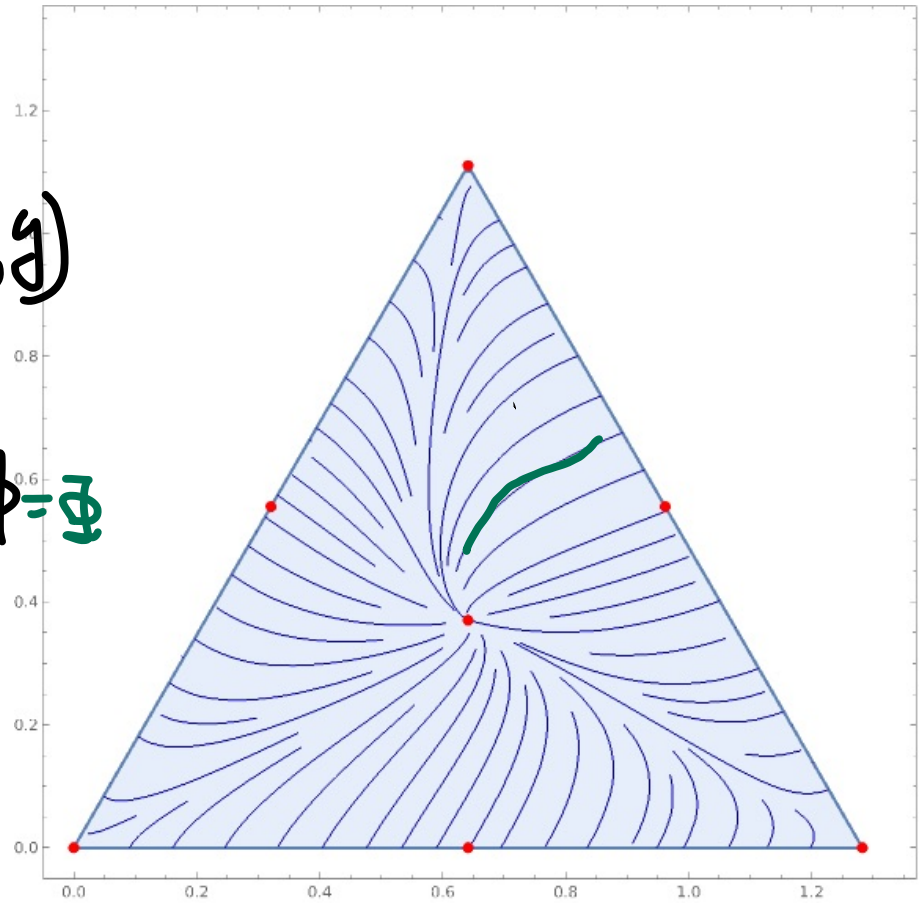
(1) If M_w is not minimal, then $T_{\max} < \infty$ and $\{M_{\xi^w(t)}\}_t$ collapses to a focal submanifold F of M_w. In addition, if M_w is irreducible, $\operatorname{codim}(M_w) \geq 2$ and the fibration $M_w \to F$ is shperical, then $\{M_{\xi^w(t)}\}_t$ has type I singularity, that is,

$$\lim_{t \to T_{\max} - 0} \max_{|v|=1} |A_v(t)|(T_{\max} - t) < \infty$$

(2) If M_w is a regular leaf of polar foliation, then $T_{\min} = \infty$ and $\{M_{\xi^w(t)}\}_{t \in [0,\infty)}$ convergences to a minimal one.

MCF/BMCF for equifocal submanifolds





integral curves of X

MCF/BMCF and Hermann actions

Principal orbits of Hermann actions are known as a model of equifocal submanifolds. We begin by recalling the notion of Hermann actions.

G/K: symmetric sp. of cpt. type with involution θ of G satisfying

$$(\mathrm{Fix}(\theta))_0 \subset K \subset \mathrm{Fix}(\theta)$$

H: a symmetric subgroup of G, i.e., $\exists\tau$: involution of G satisfying

$$(\mathrm{Fix}(\tau))_0 \subset H \subset \mathrm{Fix}(\tau)$$

Then isometric action of H on G/K is called a Hermann action and (G, θ, τ) is also called a compact symmetric triad.

MCF/BMCF and Hermann actions

Fact. Any principal orbit M of the Hermann action is an equifocal submanifold of G/K.

Remark. The converse of this fact holds if G/K: irreducible and $\mathrm{codim}(M) \geq 2$ by combining the results of Christ and Kollross.

Here, a flat section of M is given as follows:

$$\mathfrak{g} \overset{\theta}{=} \mathfrak{k} \oplus \mathfrak{p} \overset{\tau}{=} \mathfrak{h} \oplus \mathfrak{q}$$

$\mathfrak{b}$: a maximal abelian subspace of $\mathfrak{p} \cap \mathfrak{q}$, $B = \exp_G(\mathfrak{b})$

Hermann proved:

$$G = KBH$$

Fact. $\Sigma := \pi(B)$ is a flat section of M.

MCF/BMCF and Hermann actions

Our study of MCF/BMCF deals some properties which invariant under the following equivalence relation $\sim$:

Definition (Matsuki)

$(G, \theta, \tau), (G, \theta', \tau')$: two compact symmetric triads

$(G, \theta, \tau) \sim (G, \theta', \tau')$

$\iff \exists \varphi \in \mathrm{Aut}(G), \exists \nu \in \mathrm{Inn}(G)$ s.t.

$$\theta' = \varphi\theta\varphi^{-1}, \quad \tau' = \nu\varphi\tau\varphi^{-1}\nu^{-1}$$

Fact. $(G, \theta, \tau) \sim (G, \theta, \tau') \implies$ their Hermann actions are isomorphic

$\implies$ their Hermann actions are orbit-equivalent, that is, $\exists \Phi$: an isometry of G/K satisfying $\Phi(H \cdot x) = H' \cdot \Phi(x)$ $(x \in G/K)$

MCF/BMCF and Hermann actions

- From this fact, we would like to choice a good representative in the isomorphism class of (G, θ, τ), for example, (G, θ, τ) satisfying $\theta\tau = \tau\theta$ if there exists.
- In fact, for such (G, θ, τ), the tangential focal data of M can be determined from the root data associated with (G, θ, τ) ($=$ symmetric triad with multiplicities due to Ikawa (2011))

MCF/BMCF and Hermann actions

Here, we briefly recall the construction of the symmetric triad with multiplicities ass. with (G, θ, τ), which is built on a maxi. abelian subsp. $\mathfrak{b}$ of $\mathfrak{p} \cap \mathfrak{q}$ or its dual space $\mathfrak{b}^*$, as follows:

$$\Delta := \{\beta \in \mathfrak{b}^* - \{0\} \mid \mathfrak{p}_\beta \neq \{0\}\}$$

$$\Delta^V := \{\beta \in \Delta \mid \mathfrak{p}_\beta \cap \mathfrak{q} \neq \{0\}\}, \quad \Delta^H := \{\beta \in \Delta \mid \mathfrak{p}_\beta \cap \mathfrak{h} \neq \{0\}\}$$

$$m_\beta^V := \dim(\mathfrak{p}_\beta \cap \mathfrak{q}), \quad m_\beta^H := \dim(\mathfrak{p}_\beta \cap \mathfrak{h}),$$

(where $\mathfrak{p}_\beta := \{X \in \mathfrak{p} \mid \mathrm{ad}(b)^2 X = -\beta(b)^2 X \, (b \in \mathfrak{b})\}$)

Fact (Ikawa). $(\Delta, \Delta^V, \Delta^H; m^V, m^H)$ satisfies the axiom of the symmetric triad with multiplicities.

MCF/BMCF and Hermann actions

The orbit space of the Hermann action is give by the closure of

$$\widetilde{C} = \left\{ b \in \mathfrak{b} \;\middle|\; \begin{array}{ll} 0 < \beta(b) < \pi & (b \in \Delta_+^V - \Delta_-^H), \\ -\pi/2 < \beta(b) < \pi/2 & (b \in \Delta_+^H - \Delta_-^V), \\ 0 < \beta(b) < \pi/2 & (b \in \Delta_+^V \cap \Delta_-^H), \end{array} \right\}$$

$M = H \cdot x$: a prin. orb. of the Hermann action ($x = \pi(\exp(Y))$, $Y \in \widetilde{C}$)

Fact (Goertsches-Thorbergsson).

$$\mathcal{PC}(M) = \{-\cot(\beta(Y))\tilde{\beta}\,(\text{multi.} = m_\beta^V) \mid \beta \in \Delta_+^V\}$$
$$\cup \{\tan(\beta(Y))\tilde{\beta}\,(\text{multi.} = m_\beta^H) \mid \beta \in \Delta_+^H\}$$
$$\cup \{0\,(\text{multi.} = \dim(\mathfrak{p} \cap \mathfrak{z}_{\mathfrak{h}}(\mathfrak{b})))\}$$

where $\tilde{\beta} = \beta \circ \exp(Y)_*^{-1}$.

MCF/BMCF and Hermann actions

It follows from Koike (2011) that

$$\mathcal{PC}(\widetilde{M}) = \{-\frac{\tilde{\beta}}{\beta(Y)+j\pi}\,(\text{multi.} = m_\beta^V) \mid \beta \in \Delta_+^V\}$$

$$\cup\,\{-\frac{\tilde{\beta}}{\beta(Y)+(j+\frac{1}{2})\pi}\,(\text{multi.} = m_\beta^H) \mid \beta \in \Delta_+^H\}$$

$$\cup\,\{0\,(\text{multi.} = \dim(\mathfrak{p} \cap \mathfrak{z}_{\mathfrak{h}}(\mathfrak{b})))\}$$

If we put $(\lambda_\beta, b_\beta) = (-\frac{\tilde{\beta}}{\beta(Y)}, \frac{\pi}{\beta(Y)})$ $(\beta \in \Delta_+^V - \Delta_+^H)$ etc., then $\mathcal{PC}(\widetilde{M})$ is rewritten as follows:

$$\mathcal{PC}(\widetilde{M}) = \{\frac{\lambda_\beta}{1+b_\beta j} \mid \beta \in \Delta_+, j \in \mathbb{Z}\}$$

and

$$\widetilde{\boldsymbol{H}} = \textstyle\sum_{\beta\in\Delta_+^V} m_\beta^V \cot(\frac{\pi}{2b_\beta})\frac{\pi}{2b_\beta} n_\beta - \sum_{\beta\in\Delta_+^H} m_\beta^H \tan(\frac{\pi}{2b_\beta})\frac{\pi}{2b_\beta} n_\beta$$

MCF/BMCF and Hermann actions

• Let us consider the case when (G, θ, τ) is a commutative simple compact symmetric triad and when M has codimension one.

• Then the corresponding symmetric triads with multiplicities determined in B.-Ikawa (arXiv:2506.02511)).

• In what follows, we consider the case when $(\Delta, \Delta^V, \Delta^H) = (BC_1, BC_1, A_1) = (\text{I-}BC_1\text{-}A_1)$.

Table: commutative simple CST with $(\Delta, \Delta^V, \Delta^H) = (\text{I-}BC_1\text{-}A_1)$

(G, K, H)	Remark
$(SU(n), S(U(1) \times U(n-1)), S(U(c) \times U(d)))$	$n = c + d$, $1 < c \leq d$
$(SO(8), U(4), U(4)')$	
$(SO(2m), SO(2) \times SO(2m-2), U(m))$	$m \geq 3$
$(Sp(n), Sp(1) \times Sp(n-1), Sp(c) \times Sp(d))$	$n = c + d$, $1 < c \leq d$

MCF/BMCF and Hermann actions

Then, for $w \in \tilde{C}$,

$$M_w \text{ is minimal } \iff \delta_\beta + \cos(2\theta_\beta^w) = 0,$$

where

$$\theta_\beta^w = \frac{\pi}{2b_\beta}(1 - \lambda_\beta(w)), \quad \delta_\beta := \frac{m_\beta^V - m_\beta^H + m_{2\beta}^V}{m_\beta^V + m_\beta^H + m_{2\beta}^V}$$

Suppose that $\xi(t)' = \widetilde{\boldsymbol{H}}^{\xi(t)}$. Then a direct calculation shows

$$(\cos(2\theta_\beta^{\xi(t)}))' = 2g_\beta(\delta_\beta + \cos(2\theta_\beta^{\xi(t)})),$$

where

$$g_\beta := (\frac{\pi}{2b_\beta})^2(m_\beta^V + m_\beta^H + m_{2\beta}^V)\|n_\beta\|^2.$$

Hence we get

$$\cos(2\theta_\beta^{\xi(t)}) + \delta_\beta = e^{2g_\beta t}(\delta_\beta + \cos(2\theta_\beta^{\xi(0)})) \qquad \cdots\cdots (\star)$$

MCF/BMCF and Hermann actions

The following is the main result of this talk.

Theorem (B.-Koike)

Suppose that $(\Delta, \Delta^V, \Delta^H)$ *is of* (I-BC_1-A_1)*-type. Then we have:*

(1) $$\frac{|A_t|^2}{|H_t|^2} < \frac{2\delta_\beta}{m_\beta^V + m_\beta^H + m_{2\beta}^V} \frac{1}{(\cos(2\theta_\beta^{\xi(0)}) + \delta_\beta)^2} e^{-4g_\beta t},$$

(2) $$\lim_{t\to-\infty} |A_t|^2 = g_\beta,$$

(3) $$\lim_{t\to-\infty} |H_t|^2 e^{-4g_\beta t} = g_\beta(m_\beta^V + m_\beta^H + m_{2\beta}^V) \frac{(\cos(2\theta^{\xi(0)}) + \delta_\beta)^2}{1-\delta_\beta^2}$$

MCF/BMCF and Hermann actions

Pf. A direct calculation shows:

$$|H_t|^2 = g_\beta(m_\beta^V + m_\beta^H + m_{2\beta}^V)\frac{(\delta_\beta + \cos 2\theta^{\xi(t)})}{1-\cos^2 2\theta^{\xi(t)}},$$

$$|A_t|^2 = g_\beta \frac{1}{1-\cos^2 2\theta^{\xi(t)}}\{1 + 2\delta_\beta \cos 2\theta^{\xi(t)} + \cos^2 2\theta^{\xi(t)}\}$$

(1) holds by Eq ($\star$).

(2) holds by the minimal condition (that is, $\lim_{t\to-\infty} \cos 2\theta^{\xi(t)} = -\delta_\beta$).

(3) $|H_t|^2 e^{-4g_\beta t}$

$$= g_\beta(m_\beta^V + m_\beta^H + m_{2\beta}^V) \cdot \frac{1}{1-\cos^2 2\theta^{\xi(t)}} \cdot (\delta_\beta + \cos 2\theta^{\xi(t)})^2 e^{-4g_\beta t}$$

Future work. Similar calculations for other types are in progress.

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Thank you for your attention.