On the relation between the enhancement factor and the Purcell factor for a nanofiber cavity

Mark Sadgrove and Ramachandrarao Yalla
Center for Photonic Innovations, The University of Electro-Communications, 1-5-1 Chofugaoka, Chofu, Japan

We consider the relation between the enhancement factor of the guided mode photon flux in a nanofiber cavity and the cavity Purcell factor. We find that the Purcell factor may be estimated from the measured enhancement factor using only analytically calculated quantities and the cavity finesse.

I. INTRODUCTION

The bad cavity of Purcell regime of cavity QED can be characterized by the cavity quality factor Q and the Purcell factor F as an alternative to the usual classicfication in terms of cavity linewidth κ, emitter linewidth γ and emitter-cavity coupling g. In many photonic crystal cavity QED nanowaveguide experiments, the emitter is embedded in the waveguide meaning that cavity induced enhancement of spontaneous emission (SE) *does not* significantly enhance the coupling into the waveguide (see, for example, [1]). In this case the Purcell factor can be calculated by taking the ratio of the measured photoluminesence (PL) intensity at the cavity resonance wavelength compared to the PL intensity at non-resonant wavelengths.

In the case of quantum dots (q-dots) in a nanofiber cavity, the q-dots are deposited on the fiber surface. In this case the bare fiber coupling is at most 22% [2] and Purcell enhancement of the q-dot emission has the effect of enhancing both the SE rate as in the above mentioned experiments *and* the coupling efficiency of SE to the nanofiber. Therefore, in the case of a nanofiber cavity, the enhancement factor E measured by the above method of taking the ratio of on-resonant to off-resonant PL intensity is not simply the Purcell factor, but rather depends on the enhancement of coupling efficiency as well.

Because of the difficulty of measuring the Purcell factor F directly in our experiments, we would like to calculate it from the measured enhancement factor E. In this note, we demonstrate that if the cavity finesse G is known, the Purcell factor may be calculated from the measured enhancement by using the results of Refs. [3, 4]. The calculation relies on the fact that the barefiber coupling efficiency may be calculated numerically [3] and that the cavity coupling is then given by a function of the bare fiber coupling and the cavity finesse [4]. We hope that these results provide a convenient way to estimate the Purcell factor for nanofiber cavities from PL spectral intensity measurements alone.

II. DEFINITIONS

We assume that a quantum emitter is present on the surface of a nanofiber for the cases of a bare nanofiber (i.e. no cavity), and a photonic crystal nanofiber cavity (PhCNC). The following Subsection provides important definitions.

A. Polarization indices

Most of the quantities we will discuss are dependent on the orientation of the dipole emitter. For all such quantities, we use a superscript to indicate the dipole orientation. Furthermore, we use three different letters as explained below:

- *Dipole polarization superscript i.* This is the most general superscript. We have $i \in \{x, y, z\}$ which specifies the dipole orientation of the emitter, with the directions defined relative to the nanofiber axis as shown in Fig. 1(a).
- *Dipole polarization superscript j.* We use this superscript in the case where the allowable polarizations are restricted to x and y i.e. $j \in \{x, y\}$.
- *Dipole polarization superscript k.* This is the complementary superscript to j which is defined by

$$k = \begin{cases} y & \text{iff} j = x, \\ x & \text{iff} j = y. \end{cases} \tag{1}$$

This definition allows compact notation for the enhancement factor as will be seen below.

B. Decay rate definitions

The following decay rates are of importance

- The total free space decay rate of the emitter is $\Gamma_0(\lambda)$,
- The total decay rate for the emitter on the bare nanofiber is $\Gamma^i_{\rm bf}(\lambda)$ and,
- The total decay rate for the emitter in $\Gamma^i_c(\lambda)$.

Each quantity is considered to be a function of the emitter wavelength λ for convenient comparison with experiments, where the emitter bandwidth is typically broad.

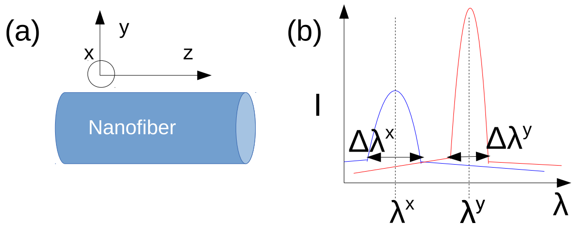

FIG. 1. (a) Definition of x, y and z axes with reference to the nanofiber. (b) Cartoon illustrating the x-mode (red) and y-mode of the PL spectrum.

The bare fiber Purcell factor $F^i_{\rm bf}$ and cavity Purcell factor F^i_c are then defined by the equations

$$\Gamma^i_{\rm bf}(\lambda) = F^i_{\rm bf}(\lambda)\Gamma_0(\lambda) \quad (2)$$
$$\Gamma^i_c(\lambda) = F^i_c(\lambda)\Gamma_0(\lambda), \quad (3)$$

where as before the i superscript of the Purcell factors denotes the dipole orientation of the emitter. To derive approximate results, it is sometimes useful to note that the bare fiber Purcell factors $F^i_{\rm bf}$ are close to unity so that

$$F^i_{\rm bf} \approx 1 \Rightarrow \Gamma^i_{\rm bf}(\lambda) \approx \Gamma_0 \;\; \forall i. \quad (4)$$

C. Photon flux definitions

We now define the following photon flux rates per unit time:

- The bare fiber photon flux per unit time is $n^i_{\rm bf}(\lambda)$ and,
- The cavity photon flux per unit time is $n^i_c(\lambda)$.

As before, these quantities depend on the emitter dipole orientation as indicated by the subscript i.

The bare fiber coupling efficiencies $\eta^i_{\rm bf}$ and the cavity coupling efficiencies η^i_c are then defined by the following equations

$$n^i_{\rm bf}(\lambda) = \eta^i_{\rm bf}(\lambda)\Gamma^i_{\rm bf}(\lambda) \quad (5)$$
$$n^i_c(\lambda) = \eta^i_c(\lambda)\Gamma^i_c(\lambda). \quad (6)$$

D. Connection of photon fluxes to the experimentally measured PL spectrum

Experimentally, for each photon emission event, the emitter is assumed to be in a random dipole orientation. Assuming no polarization filtering of the output PL, we define the following average photon fluxes:

$$n^{\rm avg}_{\rm bf}(\lambda) = \frac{1}{3}[n^x_{\rm bf}(\lambda) + n^y_{\rm bf}(\lambda) + n^z_{\rm bf}(\lambda)] \quad (7)$$
$$n^{\rm avg}_c(\lambda) = \frac{1}{3}[n^x_c(\lambda) + n^y_c(\lambda) + n^z_c(\lambda)]. \quad (8)$$

The connection to the measured PL intensities $I_{\rm bf}$ and I_c is then

$$I_{\rm bf}(\lambda) = C_{\rm bf} n^{\rm avg}_{\rm bf}(\lambda) T \quad (9)$$
$$I_c(\lambda) = C_c n^{\rm avg}_c(\lambda) T, \quad (10)$$

where T is the total measurement time and $C_{\rm bf}$ and C_c are constants which allow for any optical losses, collection efficiencies, etc of the aparatus in the case of the bare fiber and cavity respectively. For simplicity we will assume that $C_{\rm bf} = C_c$ in what follows. In this case, we may write

$$\frac{I_c(\lambda)}{I_{\rm bf}(\lambda)} \equiv \frac{n^{\rm avg}_c(\lambda)}{n^{\rm avg}_{\rm bf}(\lambda)}. \quad (11)$$

III. CALCULATION OF THE ENHANCEMENT FACTOR

Enhancement of SE occurs only when the emitter wavelength is close to the cavity resonance wavelength. However, in general, for an azimuthally asymmetric Bragg cavity structure which is typical of PhC nanofiber cavities, the x and y dipole polarizations experience different cavity resonant wavelengths. We therefore make the following definitions:

- The cavity resonant wavelength for an x oriented dipole emitter (the x cavity resonance) is λ^x,
- The cavity resonant wavelength for an y oriented dipole emitter (the y cavity resonance) is λ^y,
- The full $1/e^2$ width of the x cavity resonance is $\Delta\lambda^x$ and
- The full $1/e^2$ width of the y cavity resonance is $\Delta\lambda^y$.

These definitions are illustrated in Fig. 1(b). The significance of the $\Delta\lambda^j$ is defined as follows: For a given resonance wavelength $\lambda^j \in \{\lambda^x, \lambda^y\}$, we note that

$$n^j_c(\lambda) \approx n^j_{\rm bf}(\lambda), \;\; \text{when } |\lambda - \lambda^j| > 0.5\Delta\lambda^j. \quad (12)$$

That is, away from resonance, the photon flux into the cavity for a given dipole orientation is just equal to the bare fiber photon flux for the same emitter dipole orientation.

We define the polarization averaged enhancement factor E as the ratio of the PL intensity for an emitter in the cavity to the PL intensity for the same emitter on a bare fiber. Assuming that Eq. 11 is true for each polarization component respectively, the enhancement factor is given by

$$E(\lambda) = \frac{n^{\rm avg}_c(\lambda)}{n^{\rm avg}_{\rm bf}(\lambda)}. \quad (13)$$

In what follows, we will focus on the enhancement factor at a cavity resonance i.e. $\lambda = \lambda^j$ for $j \in \{x, y\}$.

In order proceed, we make the following assumptions:

- The emitter is located at the exact cavity center. This implies that the z orientated dipole emission does not couple to the cavity. When we apply this assumption below, we will use the polarization superscript j where necessary, to indicate that the allowable polarizations are x and y.

- $|\lambda^x - \lambda^y| \geq 0.5(\Delta\lambda^x + \Delta\lambda^y)$. Thus by Eq. 12, we have $n_c^x(\lambda^y) \approx n_{\rm bf}(\lambda^y)^x$ and $n_c^y(\lambda^x) \approx n_{\rm bf}(\lambda^x)^y$.

Then, by Eq. 12 and using the definition of the complementary polarization index in Eq. 1, we find

$$E(\lambda^j) = \frac{1}{3}\frac{n_{\rm bf}^k(\lambda^j) + n_c^j(\lambda^j)}{n_{\rm bf}^{\rm avg}(\lambda^j)}, \tag{14}$$

where we recall that k is the complementary dipole polarization index to j as defined in Section II .

Further expansion can be achieved by defining the quantity

$$N_s^i(\lambda) = \eta_s^i(\lambda)F_s^i(\lambda), \tag{15}$$

where $s \in \{{\rm bf}, c\}$ indicates whether we are considering the bare fiber or cavity values. Then from Eqs. 2 and Eq. 5 we may write

$$n_s^i(\lambda) = N_s^i(\lambda)\Gamma_0(\lambda). \tag{16}$$

It is then natural to define $n_s^{\rm avg}(\lambda) = N_s^{\rm avg}(\lambda)\Gamma_0(\lambda)$.

Because Γ_0 is a common factor in both the numerator and denominator of Eq. 14, we may rewrite it as follows:

$$E(\lambda^j) = \frac{1}{3}\frac{N_{\rm bf}^k(\lambda^i) + N_c^j(\lambda^j)}{N_{\rm bf}^{\rm avg}(\lambda^j)}. \tag{17}$$

To proceed further, we seek to expand the term $N_c^j(\lambda^j) = \eta_c^j(\lambda^j)F_c^j$. From Ref. [4], it may be shown that

$$\eta_c^i = \frac{G_c^j}{G_c^j + [\eta_{\rm bf}^j]^{-1} - 1}, \tag{18}$$

where G_c^j is the finesse of the cavity j-polarized mode resonance, where we recall that $j \in \{x, y\}$. Substituting Eq. 18 into Eq. 17, we get

$$E(\lambda^j) = \frac{1}{3}\frac{N_{\rm bf}^k(\lambda^j) + \frac{G_c^j}{G_c^j + [\eta_{\rm bf}^j]^{-1} - 1}F_c^k(\lambda^j)}{N_{\rm bf}^{\rm avg}(\lambda^j)}. \tag{19}$$

This is the most general formula for the enhancement factor which involves only the Purcell factor F_c^j, the bare fiber couplings $\eta_{\rm bf}^i$ which may be calculated from Ref. [3] and the bare fiber Purcell factors $F_{\rm bf}^i$. We can arrive at a simpler approximate formula by making the following approximation about the bare fiber Purcell factor: $F_{\rm bf}^i = F_{\rm bf}^{\rm avg}\ \forall i$. Then we may rewrite Eq. 17 as

$$E(\lambda^j) \approx \frac{\eta_{\rm bf}^k(\lambda^j) + \frac{G_c^j}{G_c^j + [\eta_{\rm bf}^j]^{-1} - 1}\frac{F_c^j(\lambda^j)}{F_{\rm bf}^{\rm avg}}}{\eta_{\rm bf}^{\rm sum}(\lambda^j)}, \tag{20}$$

where $\eta_{\rm bf}^{\rm sum}(\lambda^j) = [\eta_{\rm bf}^x(\lambda^j) + \eta_{\rm bf}^y(\lambda^j) + \eta_{\rm bf}^z(\lambda^j)]$. We note that for a 600 nm diameter nanofiber, $F_{\rm bf}^{\rm avg} = 1.2$.

Finally, since our purpose is to calculate the Purcell factor from the measured enhancement factor, we rearrange the equation for F_c^j, giving

$$F_c^j(\lambda^j) = \frac{F_{\rm bf}^{\rm avg}(\lambda^j)}{G_c^j(\lambda^j)}[E(\lambda^j)\eta_{\rm bf}^{\rm sum}(\lambda^j) - \eta_{\rm bf}^k(\lambda^j)](G_c^j + [\eta_{\rm bf}^j]^{-1} - 1). \tag{21}$$

[1] J. M. Hausmann, B. J. Shields, Q. Quan, Y. Chu, N. P. de Leon, R. Evans, M. J. Burek, A. S. Zibrov, M. Markham, D. J. Twitchen, H. Park, M. D. Lukin, and M. Loncăr, "Coupling of NV Centers to Photonic Crystal Nanobeams in Diamond", Nanoletters **13**, 57915796 (2013).

[2] R. Yalla, F. Le Kien, M. Morinaga, and K. Hakuta, "Efficient channeling of fluorescence photons from single quantum dots into guided modes of optical nanofiber," Phys. Rev. Lett. **109**, 063602 (2012).

[3] Fam Le Kien, S. Dutta Gupta, V. I. Balykin, and K. Hakuta, "Spontaneous emission of a cesium atom near a nanober: Efcient coupling of light to guided modes", Phys. Rev. A **72**, 032509 (2005).

[4] F. Le Kien and K. Hakuta, "Cavity-enhanced channeling of emission from an atom into a nanofiber," Phys. Rev. A **80**, 053826 (2009).