東京理科大学

特異点・トポロジーセミナーのお知らせ

以下の予定で，特異点・トポロジーセミナーを開催いたします．若手研究者の積極的な参加を歓迎いたします．奮ってご参加ください．

講師 ： **Marek Kaluba 氏（アダムミツキエビチ大学，ポーランド）**

日時 ： **2019 年 1 月 11 日（金），16:30〜17:30**

部屋 ： **神楽坂キャンパス，7 号館 5 階，751 セミナー室**

題目 ： **Non-commutative positivity and property (T)**

アブストラクト

The underlying theme of the talk is the reduction of the (hard analytic) question of Kazhdan's property (T) to an (algebraic, computable) problem of non-commutative optimisation. In this formulation, property (T) is equivalent to the positivity of the element $\Delta^2 - \lambda\Delta$ in (full) C^*-group algebra (where Δ is the group Laplacian). This problem will serve as a motivation to explore the topic of non-commutative positivity.

I will present certain results on positivity for free $*$-algebras and group algebras. It turns out that, due to much richer representation theory of non-commutative objects, the Positivestellenstze in the non-commutative world are much stronger than in the commutative world: the positive cone is in many cases *equal* to the cone of sums of (hermitian) squares. Moreover one of the characterisations of cones in the "algebraic topology" of $*$-algebra allows turning a numerical approximation of the the sum of squares decomposition (of $\Delta^2 - \lambda\Delta$) into a mathematical proof of the positivity.

I will finish by presenting the very concrete case of the application of the theory to $S\mathrm{Aut}(F_5)$, the (special) automorphism group of the free group on five generators. The successful execution of the described programme was obstructed by the sheer size of the optimisation problem. To overcome these difficulties we exploited the symmetry of $\Delta^2 - \lambda\Delta$ and used the theory of representations of finite groups to significantly reduce the complexity of the optimisation problem, wich might be of interest of its own.

Property (T) for $\mathrm{Aut}(F_n)$ has been a long-standing open problem. As observed by Lubotzky and Pak, the positive resolution leads to better understanding of the effectiveness of the product replacement algorithm commonly used in computational group theory to generate pseudo-random elements of finite groups.

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