Root systems of torus graphs and extended actions of torus manifolds

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A *torus manifold* is a compact, connected, oriented $2n$-dimensional T^n-manifold with fixed points (see [**HaMa**]). We can define a labelled graph from the given torus manifold as follows:

- vertices are fixed points;
- edges are invariant 2-spheres;
- edges are labelled by tangential representations around fixed points.

This labelled graph is called a *torus graph* (see [**MMP**], this may be regarded as the special class of (a little bit generalized) GKM graphs defined in [**GuZa**]).

In this talk, we study when torus actions of torus manifolds can be induced from non-abelian compact connected Lie group actions, i.e., when torus actions can be extended to non-abelian group actions (see e.g. [**Co, De, K10, K11, Ma, McTo, Wi**] for related works). To do that, we introduce *root systems of torus graphs* combinatorially (this may be regarded as the generalization of (reductive) root systems of fans in [**Co, De**] and root systems of polytopes in [**Ma**]). Using this root system, we characterize what kind of compact connected non-abelian Lie group (whose maximal torus is T^n) acts on a torus manifold. The main theorem is as follows:

Theorem 1. If a torus manifold has an extended non-abelian comapct connected Lie group G-action, then a simple factor of G is of type A, B or D.

This theorem is also proved by Wiemeler [**Wi**] by using the different method.

This is a joint work with Mikiya Masuda.

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