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Imaginary projections: Complex versus real coefficients

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Given a multivariate complex polynomial $p \in \mathbb{C}[z_1, \ldots, z_n]$, the imaginary projection $\mathcal{I}(p)$ of p is defined as the projection of the variety $\mathcal{V}(p)$ onto its imaginary part. We focus on studying the imaginary projection of complex polynomials and we state explicit results for certain families of them with arbitrarily large degree or dimension. Then, we restrict to complex conic sections and give a full characterization of their imaginary projections, which generalizes a classification for the case of real conics. That is, given a bivariate complex polynomial $p \in \mathbb{C}[z_1, z_2]$ of total degree two, we describe the number and the boundedness of the components in the complement of $\mathcal{I}(p)$ as well as their boundary curves and the spectrahedral structure of the components. We further show a realizability result for strictly convex complement components which is in sharp contrast to the case of real polynomials.

Keywords
Imaginary projection, Complex varieties, Convex algebraic geometry, Spectrahedron, Stable polynomial