

Simplification of comprehensive Gröbner systems using disequalities

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A comprehensive Gröbner system (CGS) is a powerful tool for handling parametric polynomial systems. Its first practical computation algorithm was introduced in [6]. With improvements of the subsequent works such as [3,4,5], we now have several its application programs such as the one introduced in [2].

It seems that a basic framework of its practical computation algorithm was established by the work of [5] at least from a theoretical point of view, however, there still remain many important issues concerning its efficient implementation. We have developed several techniques which improve the existing implementations of CGS. In the talk [7] of the last ACA2021, we reported that our techniques are quite effective through our implementation in SageMath [1]. Since our work was on going at that point, however, several important theoretical issues were still remained open.

In the talk, we introduce several concepts concerning simplification of CGS and settle the above open problems.

Keywords

Comprehensive Gröbner System, SageMath

References

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