

Implementation report on parametric absolute factorization of multi-variate polynomials

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This is a continuation of the author’s paper [6] where a computational strategy for *parametric polynomial ideal decomposition*, where a parametric polynomial ideal means an ideal generated by polynomials with parametric coefficients. In [6], the notion of the stability of *ideal structures*, such as radicalness and certain primality, is given by using the notion of so-called *comprehensive Gröbner bases*. Moreover, computational methods are proposed for classifying the values of parameters for such stable structures of ideals. (As references, see [1, 5] for the stability of Gröbner basis, [2, 3, 4] for computational methods of comprehensive Gröbner bases of polynomial ideals.)

Actually, for a parametric polynomial ideal, its radical computation and prime/primary decomposition, can be described uniformly and we may call them *parametric decomposition*. But, the practicality of such parametric decomposition or its efficient realization on real computer is not investigated yet.

For prime/primary decomposition, the most important and difficult part is to classify the values of parameters for the stable primality of ideals as semi-algebraic sets. And such decomposition is certainly reduced to absolutely irreducible factorization of polynomials with parametric coefficient. (Here we call it parametric factorization, in short.) Also in [6], a naive method is given for handling parametric factorization. We note that absolute irreducibility is necessary for our classification on parameter values in semi-algebraic sets. Because, irreducibility condition on parameters may not be semi-algebraic in non-algebraic closure fields. Also, as univariate polynomials can be decomposed into their linear factors in algebraic closed fields, we consider multi-variate polynomials.

In this talk, we report our current status of realization of parametric factorization based on the naive method, and give some practical improvement.

Keywords

comprehensive Gröbner system, primary decomposition

References

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