佐古研究室

佐古研究室では、非可換幾何や場の量子論の研究といった幾何や数理物理の研究と、数学教育の研究をしています。院生の研究テーマは各自の興味によって幅広い領域から自由に選んでもらいます。例えば微分幾何の最先端理論、新しい数学教育法の開発などです。また人間力育成のため、院生には国際経験を積むことを推奨し、海外渡航に可能な限り資金的援助します。

７号館４階

ゲージ理論、非可換幾何などを研究し本気で数学者や物理学者になりたい学生は大歓迎です。 詳しくは、メール等で連絡をしてから訪ねてきてください。お待ちしています。

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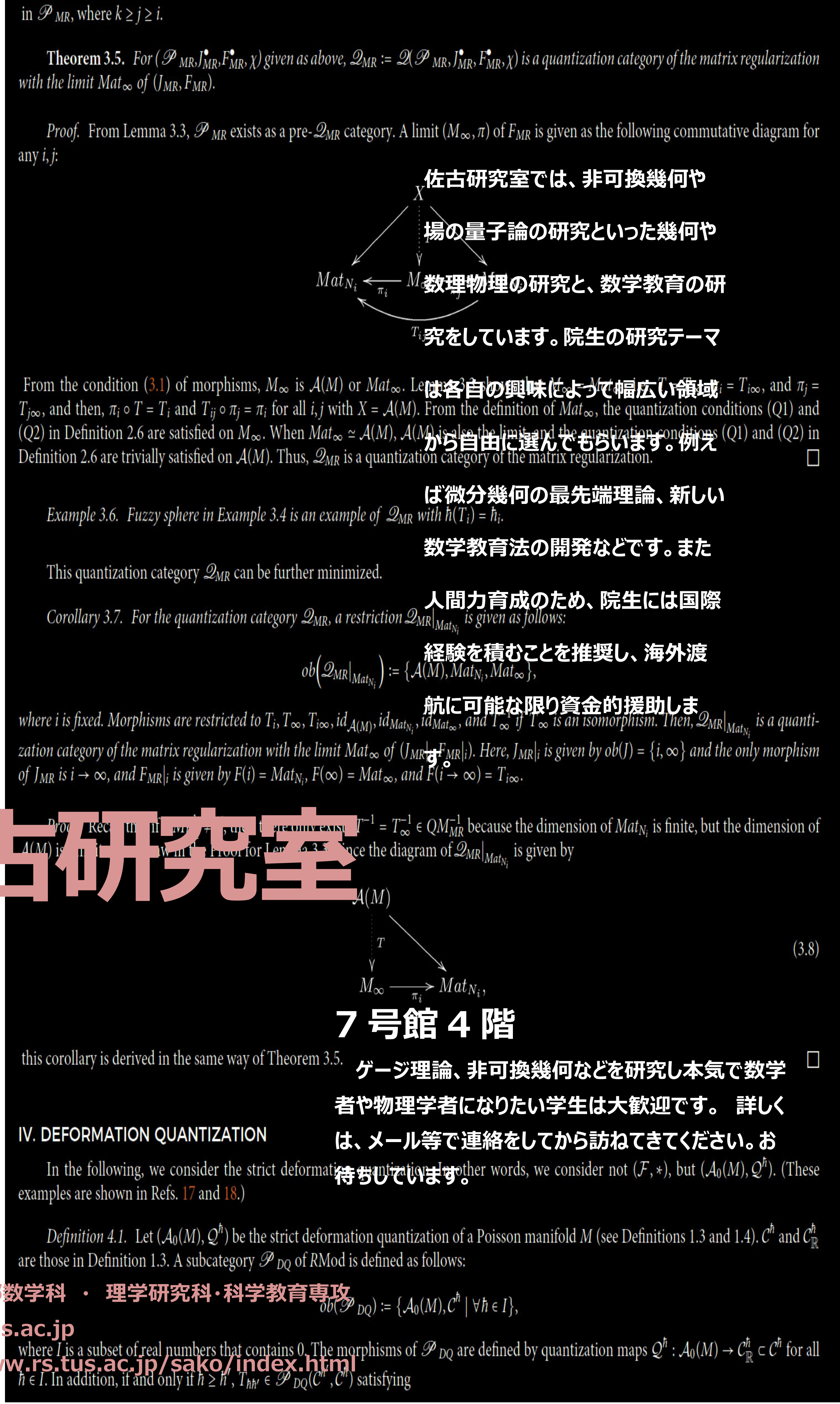

in $\mathscr{P}_{MR}$, where $k \geq j \geq i$.

Theorem 3.5. *For $(\mathscr{P}_{MR}, J^\bullet_{MR}, F^\bullet_{MR}, \chi)$ given as above, $\mathscr{Q}_{MR} := \mathscr{Q}(\mathscr{P}_{MR}, J^\bullet_{MR}, F^\bullet_{MR}, \chi)$ is a quantization category of the matrix regularization with the limit Mat_∞ of (J_{MR}, F_{MR}).*

Proof. From Lemma 3.3, $\mathscr{P}_{MR}$ exists as a pre-$\mathscr{Q}_{MR}$ category. A limit (M_∞, π) of F_{MR} is given as the following commutative diagram for any i, j:

From the condition (3.1) of morphisms, M_∞ is $\mathcal{A}(M)$ or Mat_∞. ... $= T_{i\infty}$, and $\pi_j = T_{j\infty}$, and then, $\pi_i \circ T = T_i$ and $T_{ij} \circ \pi_j = \pi_i$ for all i, j with $X = \mathcal{A}(M)$. From the definition of Mat_∞, the quantization conditions (Q1) and (Q2) in Definition 2.6 are satisfied on M_∞. When $Mat_\infty \simeq \mathcal{A}(M)$, $\mathcal{A}(M)$ is also the limit and the quantization conditions (Q1) and (Q2) in Definition 2.6 are trivially satisfied on $\mathcal{A}(M)$. Thus, $\mathscr{Q}_{MR}$ is a quantization category of the matrix regularization. □

Example 3.6. Fuzzy sphere in Example 3.4 is an example of $\mathscr{Q}_{MR}$ with $\hbar(T_i) = \hbar_i$.

This quantization category $\mathscr{Q}_{MR}$ can be further minimized.

Corollary 3.7. For the quantization category $\mathscr{Q}_{MR}$, a restriction $\mathscr{Q}_{MR}|_{Mat_{N_i}}$ is given as follows:

$$ob\left(\mathscr{Q}_{MR}|_{Mat_{N_i}}\right) := \{\mathcal{A}(M), Mat_{N_i}, Mat_\infty\},$$

where i is fixed. Morphisms are restricted to $T_i, T_\infty, T_{i\infty}, id_{\mathcal{A}(M)}, id_{Mat_{N_i}}, id_{Mat_\infty}$, and T^{-1}_∞ if T_∞ is an isomorphism. Then, $\mathscr{Q}_{MR}|_{Mat_{N_i}}$ is a quantization category of the matrix regularization with the limit Mat_∞ of $(J_{MR}|_i, F_{MR}|_i)$. Here, $J_{MR}|_i$ is given by $ob(J) = \{i, \infty\}$ and the only morphism of J_{MR} is $i \to \infty$, and $F_{MR}|_i$ is given by $F(i) = Mat_{N_i}$, $F(\infty) = Mat_\infty$, and $F(i \to \infty) = T_{i\infty}$.

... $T^{-1} = T^{-1}_\infty \in QM^{-1}_{MR}$ because the dimension of Mat_{N_i} is finite, but the dimension of ... the diagram of $\mathscr{Q}_{MR}|_{Mat_{N_i}}$ is given by

(3.8)

this corollary is derived in the same way of Theorem 3.5. □

IV. DEFORMATION QUANTIZATION

In the following, we consider the strict deformation quantization. In other words, we consider not $(\mathcal{F}, *)$, but $(\mathcal{A}_0(M), Q^\hbar)$. (These examples are shown in Refs. 17 and 18.)

Definition 4.1. Let $(\mathcal{A}_0(M), Q^\hbar)$ be the strict deformation quantization of a Poisson manifold M (see Definitions 1.3 and 1.4). $C^\hbar$ and $C^\hbar_{\mathbb{R}}$ are those in Definition 1.3. A subcategory $\mathscr{P}_{DQ}$ of RMod is defined as follows:

$$ob(\mathscr{P}_{DQ}) := \{\mathcal{A}_0(M), C^\hbar \mid \forall \hbar \in I\},$$

where I is a subset of real numbers that contains 0. The morphisms of $\mathscr{P}_{DQ}$ are defined by quantization maps $Q^\hbar : \mathcal{A}_0(M) \to C^\hbar_{\mathbb{R}} \subset C^\hbar$ for all $\hbar \in I$. In addition, if and only if $\hbar \geq \hbar'$, $T_{\hbar\hbar'} \in \mathscr{P}_{DQ}(C^\hbar, C^{\hbar'})$ satisfying