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On super mean labeling of some graphs

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Abstract. Let G be a (p,q)-graph and $f : V(G) \to \{k, k+1, k+2, k+3, \ldots, p+q+k-1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. Then f is called a k-super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, k+2, \ldots, p+q+k-1\}$. A graph that admits a k-super mean labeling is called k-super mean graph. In this paper, we present k-super mean labeling of $C_{2n} (n \neq 2)$ and super mean labeling of Double cycle $C(m,n)$, Dumb bell graph $D(m,n)$ and Quadrilateral snake Q_n.

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§1. Introduction

By a graph we mean a finite, simple and undirected one. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. The disjoint union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

Let C_m and C_n be two disjoint cycles with $u \in V(C_m)$ and $v \in V(C_n)$. The double cycle, denoted by $C(m,n)$, is the graph obtained by identifying u and v. The dumb bell graph $D(m,n)$ is obtained by joining the two vertices u and v with an edge.

The antiprism graph G on $2n$ vertices has the vertex set $\{u_i, v_i : 1 \leq i \leq n\}$ and the edge set $\{u_iu_{i+1}, v_iv_{i+1}, u_1u_n, v_1v_n : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\} \cup \{v_iu_{i-1}, v_1u_n : 2 \leq i \leq n\}$.

Any quadrilateral snake Q_n is obtained from a path $u_1u_2u_3 \ldots u_n$ by joining u_i and u_{i+1} to new vertices v_i and $w_i (1 \leq i \leq n-1)$ respectively and joining v_i to $w_i (1 \leq i \leq n-1)$. That is, every edge of the path is replaced by the cycle C_4. $\lceil x \rceil$ denotes the smallest integer greater than or equal to x. For notations and terminology we follow [2].

§2. Preliminary Results

The concept of super mean labeling was introduced in [6] and further discussed in [3, 4, 5]. B. Gayathri et al. extended the notion of k-super mean labeling of graphs [1]. Let G be a (p, q)-graph and $f : V(G) \to \{k, k+1, k+2, k+3, \ldots, p+q+k-1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. Then f is called a k-super mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, k+2, \ldots, p+q+k-1\}$. A graph that admits a k-super mean labeling is called k-super mean graph. We use the following results in the subsequent theorems.

Theorem 2.1. [6] *Any path P_n is a super mean graph.*

Theorem 2.2. [6] *Let $G_1 = (p_1, q_1)$ and $G_2 = (p_2, q_2)$ be two super mean graphs with super mean labeling f and g respectively. Let $f(u) = p_1 + q_1$ and $g(v) = 1$. Then the graph $(G_1)_f * (G_2)_g$ obtained from G_1 and G_2 by identifying the vertices u and v is also a super mean graph.*

Theorem 2.3. [6] *Any odd cycle C_{2n+1} is a super mean graph.*

Remark 2.4. [6] *C_4 is not a super mean graph.*

§3. k-Super Mean Graph

In this section we establish k-super mean labeling of the graphs such as even cycle (except C_4), antiprism on $2n$ vertices ($n > 4$), the generalized prism $C_n \times P_m$ (n is odd) and the grid $P_m \times P_n$ with one random crossing edge in every square.

Theorem 3.1. *Any even cycle $C_{2n} (n \neq 2)$ is a k-super mean graph.*

Proof. Let $V(C_{2n}) = \{u_1, u_2, u_3, \ldots, u_{2n}\}$.

For $n \neq 2$, define $f : V(C_{2n}) \to \{k, k+1, k+2, k+3, \ldots, p+q+k-1 = 4n+k-1\}$ by

$$\begin{aligned}
f(u_1) &= k, \\
f(u_2) &= k+2, \\
f(u_3) &= k+6, \\
f(u_4) &= k+11, \\
f(u_{4+i}) &= k+11+4i \text{ for } 1 \leq i \leq n-3, \\
f(u_{n+1+i}) &= 4(n-i)k \text{ for } 1 \leq i \leq n-3, \\
f(u_{2n-1}) &= k+8, \\
f(u_{2n}) &= k+5.
\end{aligned}$$

Then $f(V) = \{k, k+2, k+5, k+6, k+8, k+11, k+12, k+15, k+16, \ldots, k+4n-9, k+4n-8, k+4n-5, k+4n-4, k+4n-1\}$ and $\{f^*(e) : e \in E(C_{2n})\} = \{k+1, k+3, k+4, k+7, k+9, k+13, k+14, \ldots, k+4n-7, k+4n-6, \ldots, k+4n-3, k+4n-2\}$. Clearly $f(V) \cup \{f^*(e) : e \in E(C_{2n})\} = \{k, k+1, k+2, \ldots, k+4n-1\}$. So f is a k-super mean labeling. Hence $C_{2n} (n \neq 2)$ is a k-super mean graph. □

Example 3.2. *The 5-super mean labeling of C_8 is given in Figure 1.*

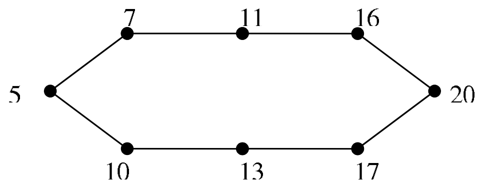


Figure 1

Theorem 3.3. *An antiprism G on $2n$ vertices $(n > 4)$ is a k-super mean graph.*

Proof. Let $\{u_i, v_i : 1 \leq i \leq n\}$ be the $2n$ vertices of the antiprism graph G.
Case (i) n is odd. Take $n = 2s + 1$.

Define $f : V(G) \to \{k, k+1, k+2, k+3, \ldots, p+q+k-1 = 6n+k-1\}$ by

$$
\begin{aligned}
f(u_1) &= k; \\
f(u_2) &= k + 5; \\
f(u_{2+i}) &= k + 5 + 4i \text{ for } 1 \leq i \leq s-1; \\
f(u_{s+2}) &= k + 4s - 2; \\
f(u_{s+2+i}) &= k + 4s - 2 - 4i \text{ for } 1 \leq i \leq s-1; \\
f(v_1) &= k + 8s + 4; \\
f(v_2) &= k + 8s + 9; \\
f(v_{2+i}) &= k + 8s + 9 + 4i \text{ for } 1 \leq i \leq s-1; \\
f(v_{s+2}) &= k + 12s + 2; \\
f(v_{s+2+i}) &= k + 12s + 2 - 4i \text{ for } 1 \leq i \leq s-1.
\end{aligned}
$$

It can be verified that f is a k-super mean labeling of G.
Case (ii) n is even. Take $n = 2s$.

Define $f : V(G) \to \{k, k+1, k+2, k+3, \ldots, p+q+k-1 = 6n+k-1\}$ by

$$\begin{aligned} f(u_1) &= k; \\ f(u_2) &= k+2; \\ f(u_3) &= k+6; \\ f(u_4) &= k+11; \\ f(u_{4+i}) &= k+11+4i \text{ for } 1 \le i \le s-3; \\ f(u_{s+2}) &= k+4s-4; \\ f(u_{s+2+i}) &= k+4s-4-4i \text{ for } 1 \le i \le s-3; \\ f(u_{2s}) &= k+5; \\ f(v_1) &= k+8s+5; \\ f(v_2) &= k+8s; \\ f(v_3) &= k+8s+2; \\ f(v_4) &= k+8s+6; \\ f(v_5) &= k+8s+11; \\ f(v_{5+i}) &= k+8s+11+4i \text{ for } 1 \le i \le s-3; \\ f(v_{s+3}) &= k+12s-4; \\ f(v_{s+3+i}) &= k+12s-4-4i \text{ for } 1 \le i \le s-3. \end{aligned}$$

Clearly the induced edge labels are distinct. Therefore f is a k-super mean labeling of G. Hence G is a k-super mean graph. □

Example 3.4. *The 3-super mean labeling of antiprism on 12 vertices is given in Figure 2.*

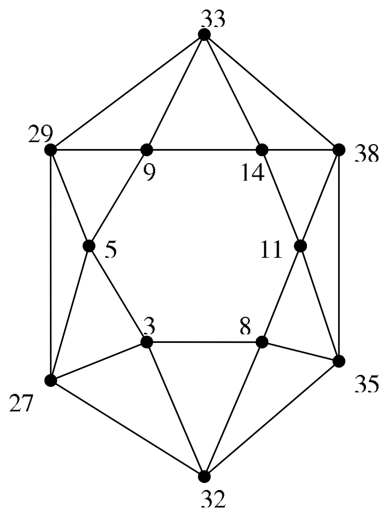


Figure 2

Theorem 3.5. *The graph $C_n \times P_m$ is a k-super mean graph where n is an odd integer and m is any integer.*

Proof. Let $\{u_j^i : 1 \le j \le n, 1 \le i \le m\}$ be the vertices of $C_n \times P_m$. Take $n = 2s + 1$.

Define $f : V(C_n \times P_m) \to \{k, k+1, k+2, k+3, \dots, p+q+k-1 = n(3m-1)+k-1\}$ by

$$\begin{aligned} f(u_j^1) &= k + 2j - 2 \text{ for } 1 \le j \le s+1; \\ f(u_{s+2}^1) &= k + 2s + 3; \\ f(u_{s+2+j}^1) &= k + 2s + 3 + 2j \text{ for } 1 \le j \le s-1; \\ f(u_1^2) &= k + 8s + 3; \\ f(u_{1+j}^2) &= k + 8s + 4 + 2j \text{ for } 1 \le j \le s; \\ f(u_{s+2}^2) &= k + 6s + 3; \\ f(u_{s+2+j}^2) &= k + 6s + 3 + 2j \text{ for } 1 \le j \le s-1. \end{aligned}$$

For $m > 2, f(u_j^m) = f(u_j^{m-2}) + 6n$ for $1 \le j \le n$. One can prove that f is a k-super mean labeling of $C_n \times P_m$. Hence the theorem. □

Example 3.6. *The 4-super mean labeling of $C_7 \times P_4$ is give in Figure 3.*

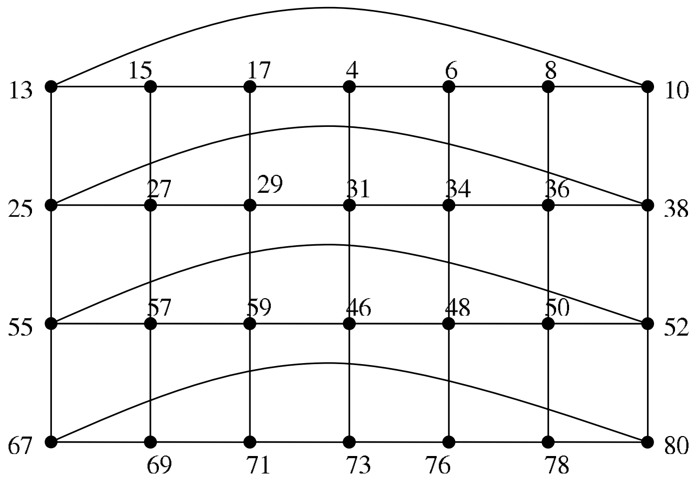


Figure 3

Theorem 3.7. *The grid $P_m \times P_n$ with one random crossing edge in every square is a k-super mean graph.*

Proof. Let $\{u_i^j : 1 \le j \le m, 1 \le i \le n\}$ be the vertices of $P_m \times P_n$. Define f as follows: $f(u_i^j) = k+2j-2+(2i-2)(2m-1)$ for all $1 \le j \le m, 1 \le i \le n$. Hence the edges $u_i^j u_{i+1}^j$ will get the label $k+2j-2+(2i-1)(2m-1)$ and the edge $u_i^j u_i^{j+1}$ will get the label $k+2j-1+(2i-2)(2m-1)$. A crossing edge is either $u_i^j u_{i+1}^{j+1}$ or $u_{i+1}^j u_i^{j+1}$ and both will get the label $k+2j-1+(2i-1)(2m-1)$. Clearly f is a k-super mean labeling. Hence the grid $P_m \times P_n$ with one random crossing edge in every square is a k-super mean graph. □

Example 3.8. *The 2-super mean labeling obtained from $P_3 \times P_4$ is given in Figure 4.*

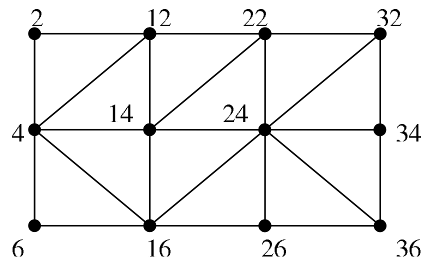


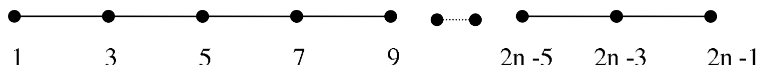
Figure 4

Note 3.9. *The k-super mean labeling of the graph G is the generalization of super mean labeling of G.*

§4. Super Mean Graph

Theorem 4.1. *Let $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ be two super mean graphs with $u \in V(G_1)$ has the label p_1+q_1 and $v \in V(G_2)$ has the label 1. Then the graph G which is obtained by joining u to v by any path P_n is a super mean graph.*

Proof. Let f and h be the super mean labelings of G_1 and G_2 respectively. Let $u_1, u_2, u_3, \ldots, u_n$ be vertices of path P_n. By Theorem 2.1, P_n is a super mean graph. Let g be the super mean labeling of P_n as follows.



Then $g(u_1) = 1$ and $g(u_n) = 2n-1$. By Theorem 2.2, $(G_1)_f * (P_n)_g = G_3$ (say) is a super mean graph. Let k be the super mean labeling of G_3. Again by Theorem 2.2, $(G_3)_k * (G_2)_h = G$ is a super mean graph. Hence G is a super mean graph. □

Theorem 4.2. *The double cycle $C(m, n)$ is a super mean graph for all $m \geq 3$ and $n \geq 3$.*

Proof. **Case (i)** $m \neq 4$ and $n \neq 4$.

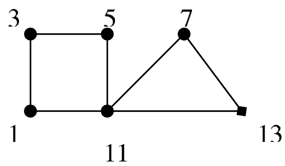
Since all cycles except C_4 are super mean graphs, by Theorem 2.2, $C(m,n)$ is a super mean graph.

Case (ii) At least one of m, n is 4. Assume $m = 4$.

Let u_1, u_2, u_3, u_4 be the vertices of C_4 and $V(C_n) = \{v_i : 1 \leq i \leq n\}$. Identify u_4 and v_1. Then $V(C(m,n)) = \{u_i, v_j : 1 \leq i \leq 4, 1 \leq j \leq n$ with $u_4 = v_1\}$.

Subcase (i) n is odd. Take $n = 2s + 1$.

A super mean labeling of $C(4,3)$ is given by



For $n > 3$, define $f : V(C(4,n)) \to \{1, 2, 3, \ldots, p + q = 2n + 7 = 4s + 9\}$ by

$$\begin{aligned}
f(u_1) &= 1;\\
f(u_2) &= 3;\\
f(u_3) &= 5;\\
f(u_4) &= f(v_1) = 11;\\
f(v_2) &= 7;\\
f(v_3) &= 12;\\
f(v_4) &= 4s + 9;\\
f(v_{4+i}) &= 2(2s - i) + 9 \text{ for } 1 \leq i \leq s - 2;\\
f(v_{s+2+i}) &= 2(4 - i) + n + 3 \text{ for } 1 \leq i \leq s - 1.
\end{aligned}$$

It can be established that f is a super mean labeling.

Subcase (ii) n is even. Take $n = 2s$.

Define $f : V(C(4,n)) \to \{1, 2, 3, \ldots, p + q = 2n + 7 = 4s + 7\}$ by

$$\begin{aligned}
f(u_1) &= 1;\\
f(u_2) &= 3;\\
f(u_3) &= 5;\\
f(u_4) &= f(v_1) = 11;\\
f(v_2) &= 7;\\
f(v_3) &= 12;\\
f(v_{3+i}) &= 12 + 2i \text{ for } 1 \leq i \leq s - 2;\\
f(v_{s+1+i}) &= 2s + 2i + 9 \text{ for } 1 \leq i \leq s - 1.
\end{aligned}$$

It can be verified that f is a super mean labeling. Hence the double cycles $C(m,n)$ are super mean graphs for all $m \geq 3$ and $n \geq 3$. □

Example 4.3. *The super mean labeling of* $C(4,8)$ *is given in Figure 5.*

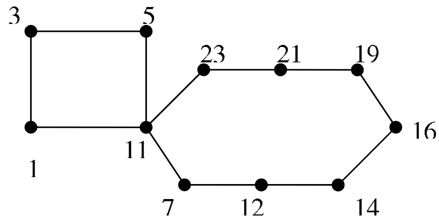


Figure 5

Theorem 4.4. *The dumb bell graph* $D(m,n)$ *is a super mean graph for all* $m \geq 3$ *and* $n \geq 3$.

Proof. We consider the following two cases.

Case (i) $m \neq 4$ and $n \neq 4$.

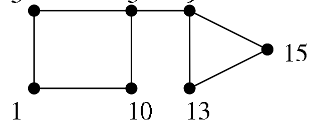
The proof follows from fact that all cycles except C_4 are super mean graphs and by Theorem 4.1.

Case (ii) At least one of m, n is 4. Let $m = 4$.

Let $V(C_m) = \{u_i : i = 1, 2, 3, 4\}$ and $V(C_n) = \{v_i : 1 \leq i \leq n\}$.

Subcase (i) n is odd. Take $n = 2s + 1$.

Join u_3 and v_3 by an edge. Then $V(D(m,n)) = V(C_m) \cup V(C_n)$ and $E(D(m,n)) = E(C_m) \cup E(C_n) \cup \{u_3v_3\}$. A super mean labeling of $D(4,3)$ is given below:



For $n > 3$, define $f : V(D(m,n)) \to \{1, 2, 3, \ldots, p + q = 2n + 9 = 4s + 11\}$ by

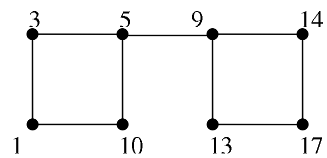
$$\begin{aligned}
f(u_1) &= 1;\\
f(u_2) &= 3;\\
f(u_3) &= 5;\\
f(u_4) &= 10;\\
f(v_1) &= 15;\\
f(v_2) &= 12;\\
f(v_3) &= 9;\\
f(v_4) &= 16;\\
f(v_{4+i}) &= 16 + 2i \text{ for } 1 \le i \le s - 2;\\
f(v_{s+3}) &= 2s + 15;
\end{aligned}$$

$$f(v_{s+3+i}) = 2s + 15 + 2i \text{ for } 1 \le i \le s - 2.$$

One can verify that f is a super mean labeling.

Subcase (ii) n is even. Take $n = 2s$.

Join u_3 and v_2 with an edge. Then $V(D(m,n)) = V(C_m) \cup V(C_n)$ and $E(D(m,n)) = E(C_m) \cup E(C_n) \cup \{u_3v_2\}$. For $n = 4$, a super mean labeling of $D(4,n)$ is given by



For $n > 4$, define $f : V(D(m,n)) \to \{1, 2, 3, \ldots, p + q = 2n + 9 = 4s + 9\}$ by

$$\begin{aligned}
f(u_1) &= 1;\\
f(u_2) &= 3;\\
f(u_3) &= 5;\\
f(u_4) &= 10;\\
f(v_1) &= 13;\\
f(v_2) &= 9;\\
f(v_3) &= 14;\\
f(v_{3+i}) &= 14 + 2i \text{ for } 1 \le i \le s - 2;\\
f(v_{s+2}) &= 2s + 13;\\
f(v_{s+2+i}) &= 2s + 13 + 2i \text{ for } 1 \le i \le s - 2.
\end{aligned}$$

It can be established that f is a super mean labeling. Hence the dumb bell graphs $D(m,n)$ are super mean graphs for all $m \geq 3$ and $n \geq 3$. □

Example 4.5. *The super mean labeling of $D(4,7)$ is given in Figure 6.*

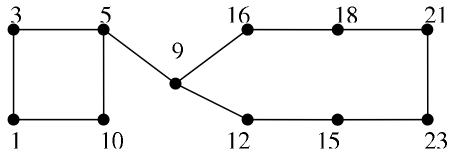


Figure 6

Theorem 4.6. *Let $C_n(n \geq 3)$ be an odd cycle. Consider n copies of an odd cycle $C_m(m \geq 3)$. If G is a graph obtained by identifying a vertex of each cycle C_m with a vertex of the cycle C_n is a super mean graph.*

Proof. Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of the cycle C_n. Let $u_{1j}, u_{2j}, u_{3j}, \ldots, u_{nj}$, $1 \leq j \leq m$, be the vertices of the cycles $C_m^{(1)}, C_m^{(2)}, C_m^{(3)}, \ldots, C_m^{(n)}$ respectively, identified at each vertex of C_n such that $u_1 = u_{1m}, u_2 = u_{21}, u_3 = u_{3m}, \ldots, u_{n-1} = u_{n-1,1}$ and $u_n = u_{nm}$ which means that $u_{1m}, u_{21}, u_{3m}, u_{41}, \ldots, u_{n-1,1}, u_{nm}$ are the vertices of the cycle C_n.

Take $n = 2s+1$ and $m = 2t+1$.

Define $f : V(G) \to \{1, 2, 3, \ldots, (2m+1)n = 8st + 6s + 4t + 3\}$ as follows:

For the cycle $C_m^{(1)}$, $f(u_{1j}) = \begin{cases} 2j-1 & \text{for } 1 \leq j \leq t+1 \\ 2j & \text{for } t+2 \leq j \leq m. \end{cases}$

For the cycle $C_m^{(k)}$, where $2 \leq k \leq s+1$,

$$f(u_{kj}) = \begin{cases} 2(k-1)m + 2(j-1) + k & \text{for } 1 \leq j \leq t+1 \\ 2(k-1)m + 2(j-1) + k + 1 & \text{for } t+2 \leq j \leq m. \end{cases}$$

For the cycle $C_m^{(k)}$, where $s+2 \leq k \leq n$.

$$f(u_{kj}) = \begin{cases} 2(k-1)m + 2(j-1) + k + 1 & \text{for } 1 \leq j \leq t+1 \\ 2(k-1)m + 2(j-1) + k + 2 & \text{for } t+2 \leq j \leq m. \end{cases}$$

Now we have $\bigcup_{i=1}^{n} \{f(V(C_m^{(i)})) \cup f^*(E(C_m^{(i)}))\} = \{1, 2, 3, \ldots, 2m\} \cup \{2m+2, 2m+3, \ldots, 4m+1\} \cup \{4m+3, 4m+4, \ldots, 6m+2\} \cup \cdots \cup \{(2m+1)s+1, (2m+1)s+2, \ldots, (2m+1)s+2m\} \cup \{(2m+1)(s+1)+2, (2m+1)(s+1)+3, \ldots, (2m+1)(s+2)\} \cup \cdots \cup \{(2m+1)(n-1)+2, \ldots, (2m+1)n\}$. Clearly these labels are all distinct. Further the labels of the edges $u_1u_2, u_2u_3, u_3u_4, \ldots, u_{s+1}u_{s+2}, u_{s+2}u_{s+3}, \ldots, u_nu_1$ of the cycle C_n are $2m+1, 4m+2, 6m+3, \ldots, (2m+1)(s+1)+1, (2m+1)(s+2)+1 \ldots (2m+1)(s+1)$ respectively. It can be easily verified that $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \ldots, n(2m+1)\}$. Hence G is a super mean graph. □

Corollary 4.7. *The graph $C_{2n+1} \odot K_2$ is a super mean graph for all n.*

Example 4.8. *The super mean labeling of G obtained from C_3 by identifying a vertex of the cycle C_5 with each vertex of the cycle C_3 is given in Figure 7.*

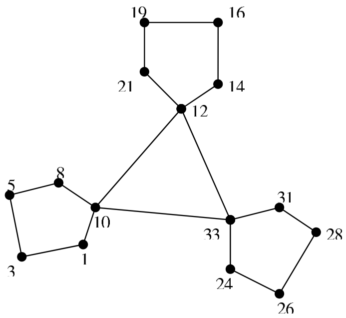


Figure 7

The graph Q_2 is C_4, and hence it is not a super mean graph [6]. Next we prove Q_n is a super mean graph for all odd values of n.

Theorem 4.9. *The quadrilateral snake Q_n, where n is odd, is a super mean graph.*

Proof. Let $V(Q_n) = \{u_i, v_i, w_i, u_n : 1 \leq i \leq n-1\}$.

Define $f : V(Q_n) \to \{1, 2, 3, \ldots, 7n-6\}$ by

$$
\begin{aligned}
f(u_1) &= 1; \\
f(u_{2i}) &= f(u_{2i-1}) + 10 \text{ for } 1 \leq i \leq s; \\
f(u_{2i+1}) &= f(u_{2i}) + 4 \text{ for } 1 \leq i \leq s; \\
f(v_1) &= 3; \\
f(v_{2i}) &= f(v_{2i-1}) + 4 \text{ for } 1 \leq i \leq s; \\
f(v_{2i+1}) &= f(v_{2i}) + 10 \text{ for } 1 \leq i \leq s-1; \\
f(w_1) &= 5; \\
f(w_{i+1}) &= f(w_i) + 7 \text{ for } 1 \leq i \leq n-1.
\end{aligned}
$$

Clearly $f(V) \cup \{f^*(e) : e \in E(Q_n)\} = \{1, 2, 3, \ldots, 7n-6\}$. Hence, Q_n where n is odd, is a super mean graph. □

Example 4.10. *The super mean labelig of Q_5 is given in Figure 8.*

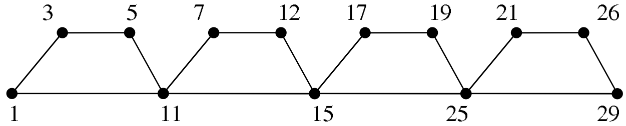


Figure 8

Theorem 4.11. *Let $C_n : u_1u_2u_3 \ldots u_nu_1$ (n is odd) be a cycle. Let G be the graph with $V(G) = V(C_n) \cup \{v_i : 1 \leq i \leq n\}, E(G) = E(C_n) \cup \{u_iv_i, u_{i+1}v_i : 1 \leq i \leq n-1\} \cup \{u_nv_n, u_1v_n\}$. Then G is a super mean graph.*

Proof. Take $n = 2s + 1$. Define $f : V(G) \to \{1, 2, 3, \ldots, p + q = 5n\}$ by

$$\begin{aligned} f(u_1) &= 1; \\ f(u_i) &= 5i - 4 \text{ for } 2 \leq i \leq s + 1; \\ f(u_{s+2}) &= 5s + 8; \\ f(u_{s+2+i}) &= 5s + 8 + 5i \text{ for } 1 \leq i \leq s - 1; \\ f(v_1) &= 3; \\ f(v_i) &= 5i - 2 \text{ for } 2 \leq i \leq s; \\ f(v_{s+1}) &= 5s + 6; \\ f(v_{s+2}) &= 5(s + 2); \\ f(u_{s+2+i}) &= 5(s + 2) + 5i \text{ for } 1 \leq i \leq s - 1. \end{aligned}$$

Clearly the vertex labels, the induced edge labels are distinct and $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \ldots, 5n\}$. Hence G is a super mean graph. □

Theorem 4.12. *Let $C_n : u_1u_2u_3 \ldots u_nu_1$ (n is odd) be a cycle. Let G be the graph obtained from C_n by joining the vertices u_i and u_{i+1} by the path P^i_m (m is odd) $1 \leq i \leq n-1$ and joining the vertices u_n and u_1 by the path P^n_m. Then G is a super mean graph.*

Proof. By Theorem 4.11, the theorem is true when $m = 3$. We prove the theorem for $m > 3$. Let $v^j_1, v^j_2, v^j_3, \ldots, v^j_m$ for $1 \leq j \leq m$ be the vertices of the path $P^i_m (1 \leq i \leq n)$ such that $v^j_m = v^{j+1}_1 = u_{j+1}$ for $1 \leq j \leq n-1$ and $v^n_m = v^1_1 = u_1$. Take $n = 2s + 1$ and $m = 2t + 1$.

Define $f : V(G) \to \{1, 2, 3, \ldots, p + q = n(2m - 1)\}$ by

$$\begin{aligned} f(v^1_i) &= 2i - 1 \text{ for } 1 \leq i \leq t + 1; \\ f(v^1_i) &= 2i \text{ for } t + 2 \leq i \leq 2t + 1; \\ f(v^j_i) &= f(v^{j-1}_i) + 2m - 1 \text{ for } 1 \leq i \leq 2t + 1 \text{ and } 2 \leq j \leq s; \\ f(v^{s+1}_1) &= f(v^s_m) = 1 + (2m - 1)s; \\ f(v^{s+1}_2) &= 4 + (2m - 1)s; \end{aligned}$$

$$f(v_{2+i}^{s+1}) = 4 + (2m-1)s + 2i \text{ for } 1 \leq i \leq t-2;$$
$$f(v_{t+1}^{s+1}) = 2t(2s+1) + s + 4;$$
$$f(v_{t+1+i}^{s+1}) = 2t(2s+1) + s + 4 + 2i \text{ for } 1 \leq i \leq t;$$
$$f(v_i^{s+2}) = 4t(s+1) + s + 2 + 2i \text{ for } 1 \leq i \leq t+1;$$
$$f(v_i^{s+2}) = 4t(s+1) + s + 3 + 2i \text{ for } t+2 \leq i \leq 2t+1;$$
$$f(v_i^j) = f(v_i^{j-1}) + 2m - 1 \text{ for } 1 \leq i \leq 2t+1 \text{ and } s+3 \leq j \leq 2s;$$
$$f(v_{1+i}^{2s+1}) = f(v_m^{2s}) + 2i \text{ for } 1 \leq i \leq 2t-1.$$

It can be verified that f is a super mean labeling of G. Hence G is a super mean graph. □

Example 4.13. *The super mean labeling of G with $m = 5$ and $n = 7$ is given in Figure 9.*

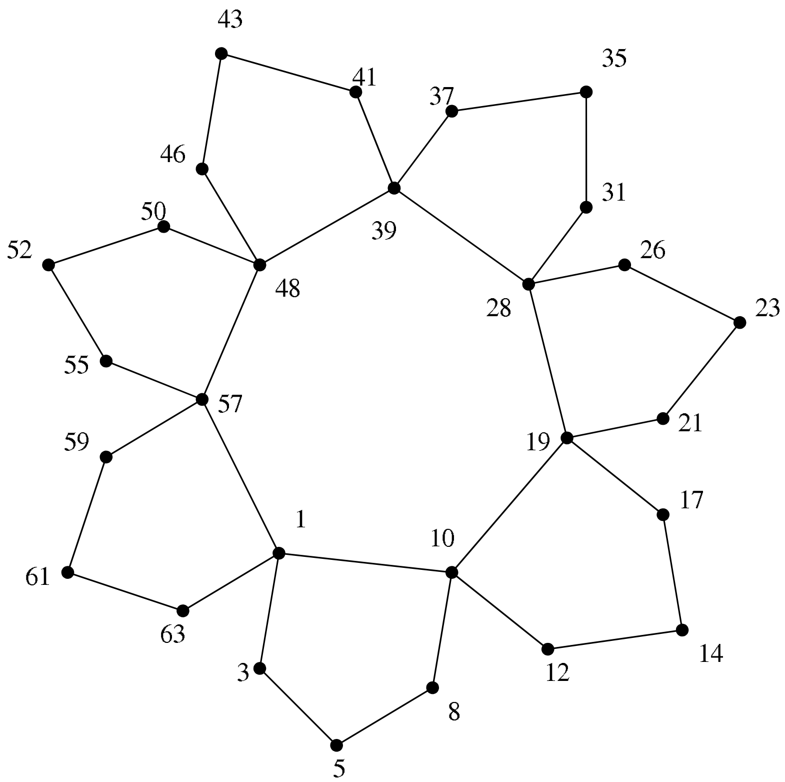


Figure 9

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