

3-product cordial labeling

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Abstract. A mapping $f : V(G) \rightarrow \{0, 1, 2\}$ is called a 3-product cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for any $i, j \in \{0, 1, 2\}$, where $v_f(i)$ denotes the number of vertices labeled with i , $e_f(i)$ denotes the number of edges xy with $f(x)f(y) \equiv i \pmod{3}$. A graph with a 3-product cordial labeling is called a 3-product cordial graph. In this paper, we investigate the 3-product cordial behaviour for some standard graphs.

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§1. Introduction

By a graph we mean finite, simple and undirected one. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ so that the order and size of G are $|V(G)|$ and $|E(G)|$ respectively. The union $G_1 \cup G_2$ of two graphs G_1 and G_2 has the vertex set $V(G_1) \cup V(G_2)$ and the edge set $E(G_1) \cup E(G_2)$. A corona $G_1 \circ G_2$ is a graph obtained from two graphs G_1 and G_2 by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining by an edge the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2 . The graph $\langle B_{n,n} : w \rangle$ is obtained by joining the center vertices of two $K_{1,n}$ stars to a new vertex w . The flower graph Fl_n is the graph obtained from a helm graph H_n by joining each pendant vertex to the apex vertex of the helm. The complete survey of graph labeling is in [2].

Cordial labeling is a weaker version of graceful labeling and harmonious labeling introduced by I. Cahit in [1]. Let f be a function from the vertices of G to $\{0, 1\}$ and for each edge xy assign the label $|f(x) - f(y)|$. f is called a cordial labeling of G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. M. Sundaram et al. introduced the concept of product cordial labeling of a

graph in [3]. A product cordial labeling of a graph G with the vertex set V is a function f from V to $\{0, 1\}$ such that if each edge uv is assigned the label $f(u)f(v)$, $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. The concept of EP -cordial labeling was introduced in [4]. A vertex labeling $f : V(G) \rightarrow \{-1, 0, 1\}$ is said to be an EP -cordial labeling if it induces the edge labeling f^* defined by $f^*(uv) = f(u)f(v)$, for each $uv \in E(G)$ and if $|v_f(i) - v_f(j)| \leq 1$ and $|e_{f^*}(i) - e_{f^*}(j)| \leq 1, i \neq j, i, j \in \{-1, 0, 1\}$, where $v_f(x)$ and $e_{f^*}(x)$ denote the number of vertices and edges of G having the label $x \in \{-1, 0, 1\}$. In [5] it is remarked that any EP -cordial labeling is a 3-product cordial labeling. A mapping $f : V(G) \rightarrow \{0, 1, 2\}$ is called a 3-product cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for any, $i, j \in \{0, 1, 2\}$, where $v_f(i)$ denotes the number of vertices labeled with i , $e_f(i)$ denotes the number of edges xy with $f(x)f(y) \equiv i \pmod{3}$. A graph with a 3-product cordial labeling is called a 3-product cordial graph.

§2. Main Results

Theorem 2.1. *If G_1 is a 3-product cordial graph with $3m$ vertices and $3n$ edges and G_2 is any 3-product cordial graph, then $G_1 \cup G_2$ is also 3-product cordial graph.*

Proof. Let f, g be a 3-product cordial labeling of G_1 and G_2 respectively. Since G_1 has $3n$ edges and $3m$ vertices we have $e_f(0) = e_f(1) = e_f(2) = n$ and $v_f(0) = v_f(1) = v_f(2) = m$. Define a labeling $h : V(G_1 \cup G_2) \rightarrow \{0, 1, 2\}$ by

$$h(u) = \begin{cases} f(u) & \text{if } u \in V(G_1) \\ g(u) & \text{if } u \in V(G_2) \end{cases}$$

Hence, $v_h(i) = v_f(i) + v_g(i)$ and $e_h(i) = e_f(i) + e_g(i)$ for $i = 0, 1, 2$. Therefore $|v_h(i) - v_h(j)| \leq 1$ and $|e_h(i) - e_h(j)| \leq 1$ for $i, j \in \{0, 1, 2\}$. Hence h is a 3-product cordial labeling of $G_1 \cup G_2$. \square

In [4], it is proved that if G is 3-product cordial graph of order p and size q then $q \leq 1 + \frac{p}{3} + \frac{p^2}{3}$ and also proved that K_n is 3-product cordial if and only if $n \leq 3$. In this paper we give an improved upper bound for q in Theorems 2.2, 2.4 and 2.6 and prove that K_n is 3-product cordial if and only if $n \leq 2$.

Theorem 2.2. *If $G(p, q)$ is a 3-product cordial graph with $p \equiv 0 \pmod{3}$, then $q \leq \frac{p^2 - 3p + 6}{3}$.*

Proof. Let f be a 3-product cordial labeling of the graph G . Take $p = 3n$. Then we have $v_f(0) = v_f(1) = v_f(2) = n$. Hence, $e_f(1) \leq n(n-1)$. Since f is a 3-product cordial labeling, $e_f(2) \leq n(n-1) + 1$ and $e_f(0) \leq n(n-1) + 1$. Hence, $q = e_f(0) + e_f(1) + e_f(2) \leq 3n(n-1) + 2 = \frac{p^2 - 3p + 6}{3}$. \square

Remark 2.3. The graph given in Figure 1 is an example for a 3-product cordial graph with $p = 6$ and $q = \frac{p^2-3p+6}{3} = 8$.

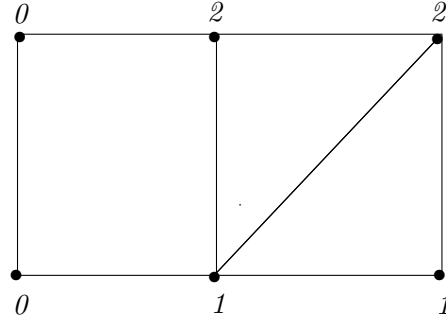


Figure 1

Theorem 2.4. If $G(p, q)$ is a 3-product cordial graph with $p \equiv 1 \pmod{3}$, then $q \leq \frac{p^2-2p+7}{3}$.

Proof. Let f be a 3-product cordial labeling of the graph G . Take $p = 3n + 1$. If $v_f(0) = v_f(2) = n$ and $v_f(1) = n + 1$, or $v_f(0) = v_f(1) = n$ and $v_f(2) = n + 1$. Then $e_f(1) \leq n^2$. If $v_f(0) = n + 1$ and $v_f(1) = v_f(2) = n$. Then $e_f(1) \leq n(n - 1)$. Thus in any case, we have $e_f(1) \leq n^2$. Since f is a 3-product cordial labeling, $e_f(2) \leq n^2 + 1$ and $e_f(0) \leq n^2 + 1$. Hence, $q = e_f(0) + e_f(1) + e_f(2) \leq 3n^2 + 2 = \frac{p^2-2p+7}{3}$. \square

Remark 2.5. The graph given in Figure 2 is an example for 3-product cordial graph with $p = 4$ and $q = \frac{p^2-2p+7}{3} = 5$.

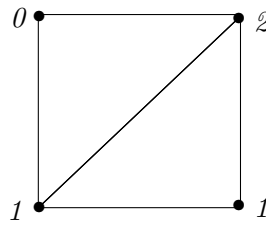


Figure 2

Theorem 2.6. If $G(p, q)$ is a 3-product cordial graph with $p \equiv 2 \pmod{3}$, then $q \leq \frac{p^2-p+4}{3}$.

Proof. Let f be a 3-product cordial labeling of the graph G . Take $p = 3n + 2$. If $v_f(0) = v_f(1) = n + 1$ and $v_f(2) = n + 1$. Hence $e_f(1) \leq n(n + 1)$. If $v_f(0) = n + 1, v_f(1) = n$ and $v_f(2) = n + 1$ or $v_f(0) = n + 1, v_f(1) = n + 1, v_f(2) = n$. Then $e_f(1) \leq n^2$. Thus in any case, we have $e_f(1) \leq n(n + 1)$. Since f is a 3-product cordial labeling, $e_f(2) \leq n(n + 1) + 1$ and $e_f(0) \leq n(n + 1) + 1$. Hence, $q = e_f(0) + e_f(1) + e_f(2) \leq 3n(n + 1) + 2 = \frac{p^2-p+4}{3}$. \square

Remark 2.7. The graph given in Figure 3 is an example for a 3-product cordial graph with $p = 5$ and $q = \frac{p^2-p+4}{3} = 8$.

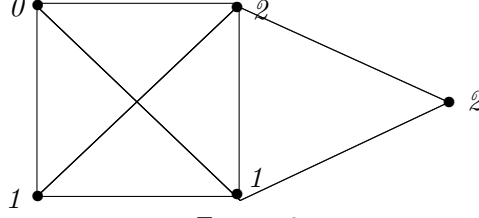


Figure 3

Theorem 2.8. $\langle B_{n,n} : w \rangle$ is a 3-product cordial graph.

Proof. Let $v_1^{(1)}, v_2^{(1)}, \dots, v_n^{(1)}$ be the pendant vertices of the star $K_{1,n}^{(1)}$ and $v_1^{(2)}, v_2^{(2)}, \dots, v_n^{(2)}$ be the pendant vertices of the star $K_{1,n}^{(2)}$. Let c_1, c_2 be the center vertices of $K_{1,n}^{(1)}, K_{1,n}^{(2)}$ respectively and they are adjacent to a new common vertex w . Let $G = \langle B_{n,n} : w \rangle$.

Case i. $n \equiv 0 \pmod{3}$. Let $n = 3k$.

Define $f : V(G) \rightarrow \{0, 1, 2\}$ by

$$\begin{aligned} f(w) &= 1, \\ f(c_1) &= f(c_2) = 1, \\ f(v_i^{(1)}) &= \begin{cases} 0 & \text{for } 1 \leq i \leq 2k+1 \\ 1 & \text{for } i > 2k+1 \end{cases} \end{aligned}$$

and

$$f(v_i^{(2)}) = \begin{cases} 2 & \text{for } 1 \leq i \leq 2k+1 \\ 1 & \text{for } i > 2k+1. \end{cases}$$

Since $v_f(0) = v_f(1) = v_f(2) = 2k+1$, $e_f(0) = e_f(2) = 2k+1$ and $e_f(1) = 2k$, we have $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| = 0$ for all $i, j = 0, 1, 2$. Thus f is a 3-product cordial labeling.

Case ii. $n \equiv 1 \pmod{3}$. Let $n = 3k+1$.

Define $f : V(G) \rightarrow \{0, 1, 2\}$ by

$$\begin{aligned} f(w) &= 1, \\ f(c_1) &= f(c_2) = 1, \\ f(v_i^{(1)}) &= \begin{cases} 0 & \text{for } 1 \leq i \leq 2k+2 \\ 1 & \text{for } i > 2k+2 \end{cases} \end{aligned}$$

and

$$f(v_i^{(2)}) = \begin{cases} 2 & \text{for } 1 \leq i \leq 2k+1 \\ 1 & \text{for } i > 2k+1. \end{cases}$$

Hence $e_f(1) = e_f(2) = 2k + 1$, $e_f(0) = 2k + 2$, $v_f(0) = v_f(1) = 2k + 2$, and $v_f(2) = 2k + 1$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$.

Case iii. $n \equiv 2 \pmod{3}$ and let $n = 3k + 2$.

Define $f : V(G) \rightarrow \{0, 1, 2\}$ by

$$\begin{aligned} f(w) &= 1, \\ f(c_1) &= f(c_2) = 1, \\ f(v_i^{(1)}) &= \begin{cases} 0 & \text{for } 1 \leq i \leq 2k + 2 \\ 1 & \text{for } i > 2k + 2 \end{cases} \end{aligned}$$

and

$$f(v_i^{(2)}) = \begin{cases} 2 & \text{for } 1 \leq i \leq 2k + 2 \\ 1 & \text{for } i > 2k + 2. \end{cases}$$

Since $e_f(0) = e_f(1) = e_f(2) = 2k + 2$, $v_f(0) = v_f(2) = 2k + 2$ and $v_f(1) = 2k + 3$, we have $|e_f(i) - e_f(j)| = 0$ and $|v_f(i) - v_f(j)| \leq 1$ for all, $i, j = 0, 1, 2$. Thus $\langle B_{n,n} : w \rangle$ is 3-product cordial. \square

§3. 3-product cordial labeling of cycle related graphs

Theorem 3.1. K_n is a 3-product cordial if and only if $n \leq 2$.

Proof. K_n has n vertices and $\frac{n(n-1)}{2}$ edges. Clearly K_1 and K_2 are 3-product cordial graphs. Conversely assume that K_n is a 3-product cordial. We consider the following three cases.

If $n \equiv 0 \pmod{3}$ then by Theorem 2.2, $\frac{n(n-1)}{2} \leq \frac{n^2-3n+6}{3}$, which implies that $n^2 + 3n - 12 \leq 0$. This is true only for $n \leq 2$.

If $n \equiv 1 \pmod{3}$ then by Theorem 2.4, $\frac{n(n-1)}{2} \leq \frac{n^2-2n+7}{3}$, which implies that $n^2 + n - 14 \leq 0$. This is true only for $n \leq 3$.

If $n \equiv 2 \pmod{3}$ then by Theorem 2.6, $\frac{n(n-1)}{2} \leq \frac{n^2-n+4}{3}$, which implies that $n^2 - n - 8 \leq 0$. This is true only for $n \leq 3$.

When $n = 3$, $K_3 = C_3$ which is not a 3-product cordial graph. Hence K_n is not a 3-product cordial graph for $n \geq 3$. \square

Theorem 3.2. $C_n \cup P_n$ is 3-product cordial for all $n \geq 3$.

Proof. Let v_1, v_2, \dots, v_n be the vertices of C_n and u_1, u_2, \dots, u_n be the vertices of P_n . Let $G = C_n \cup P_n$.

Define $f : V(G) \rightarrow \{0, 1, 2\}$ for the following three cases.

Case i. $n \equiv 0 \pmod{3}$ and let $n = 3k$.

Define f by

$$f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq 2k \\ 1 & \text{otherwise} \end{cases}$$

and

$$f(v_i) = \begin{cases} 1 & \text{for } i = 2m - 1, 1 \leq m \leq k \\ 2 & \text{otherwise.} \end{cases}$$

Hence, $v_f(0) = v_f(1) = v_f(2) = 2k$, $e_f(0) = e_f(2) = 2k$ and $e_f(1) = 2k - 1$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$.

Case ii. $n \equiv 1 \pmod{3}$ and let $n = 3k + 1$.

Define f by

$$f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq 2k \\ 1 & \text{otherwise} \end{cases}$$

and

$$f(v_i) = \begin{cases} 1 & \text{for } i = 2m - 1, 1 \leq m \leq k \\ 2 & \text{otherwise.} \end{cases}$$

Here $e_f(0) = e_f(2) = 2k$, $e_f(1) = 2k + 1$, $v_f(0) = 2k$ and $v_f(1) = v_f(2) = 2k + 1$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$.

Case iii. $n \equiv 2 \pmod{3}$ and let $n = 3k + 2$.

Define f by

$$f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq 2k + 1 \\ 1 & \text{for } 2k + 1 < i \leq 3k + 1 \\ 2 & \text{for } i = 3k + 2 \end{cases}$$

and

$$f(v_i) = \begin{cases} 1 & \text{for } i = 3k + 2, 2m - 1, 1 \leq m \leq k \\ 2 & \text{otherwise.} \end{cases}$$

Hence, $v_f(0) = v_f(1) = 2k + 1$, $v_f(2) = 2k + 2$ and $e_f(0) = e_f(1) = e_f(2) = 2k + 1$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, $C_n \cup P_n$ is 3-product cordial. \square

Theorem 3.3. $C_m \circ \overline{K_n}$ is 3-product cordial for $m \geq 3$ and $n \geq 1$.

Proof. Let $V(C_m \circ \overline{K_n}) = \{u_1, u_2, \dots, u_m, v_1, v_2, \dots, v_{mn}\}$ where u_1, u_2, \dots, u_m be the vertices of the cycle and v_1, v_2, \dots, v_{mn} be the vertices of m copies of $\overline{K_n}$. Hence, we have $|V(C_m \circ \overline{K_n})| = |E(C_m \circ \overline{K_n})| = m(n + 1)$.

Case i. $n \equiv 2 \pmod{3}$. Take $n = 3k - 1$, $m \geq 1$.

Define $f : V(C_m \circ \overline{K_n}) \rightarrow \{0, 1, 2\}$ by $f(u_i) = 1$ for $1 \leq i \leq m$;

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \leq j \leq km \\ 2 & \text{for } km < j \leq 2km \\ 1 & \text{for } j > 2km. \end{cases}$$

Here we have, $v_f(0) = v_f(1) = v_f(2) = km$, $e_f(0) = e_f(1) = e_f(2) = km$. Therefore, $|e_f(i) - e_f(j)| = 0$ and $|v_f(i) - v_f(j)| = 0$ for all $i, j = 0, 1, 2$.

Case ii. $n \equiv 0 \pmod{3}$ and let $n = 3k$. We consider the following subcases:

Subcase i. $m = 3c$.

Define f by $f(u_i) = 1$ for $1 \leq i \leq m$ and

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \leq j \leq c(3k+1) \\ 2 & \text{for } c(3k+1) < j \leq 2c(3k+1) \\ 1 & \text{for } j > 2c(3k+1). \end{cases}$$

Here we have, $v_f(0) = v_f(1) = v_f(2) = c(3k+1)$ and $e_f(0) = e_f(1) = e_f(2) = c(3k+1)$. Therefore, $|e_f(i) - e_f(j)| = 0$ and $|v_f(i) - v_f(j)| = 0$ for all $i, j = 0, 1, 2$.

Subcase ii. $m = 3c + 1$.

Define f by $f(u_i) = 1$ for $1 \leq i \leq m$ and

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \leq j \leq c(3k+1) + k \\ 2 & \text{for } c(3k+1) + k < j \leq 2c(3k+1) + 2k \\ 1 & \text{for } j > 2c(3k+1) + 2k. \end{cases}$$

Hence we have, $v_f(0) = v_f(2) = c(3k+1) + k$, $v_f(1) = c(3k+1) + k + 1$, $e_f(0) = e_f(2) = c(3k+1) + k$ and $e_f(1) = c(3k+1) + k + 1$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$.

Subcase iii. $m = 3c + 2$.

Define f by $f(u_i) = 1$ for $1 \leq i \leq m$ and

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \leq j \leq c(3k+1) + 2k + 1 \\ 2 & \text{for } c(3k+1) + 2k + 1 < j \leq 2c(3k+1) + 4k + 1 \\ 1 & \text{for } j > 2c(3k+1) + 4k + 1. \end{cases}$$

Hence we have, $v_f(0) = v_f(1) = c(3k+1) + 2k + 1$, $v_f(2) = c(3k+1) + 2k$, $e_f(0) = e_f(1) = c(3k+1) + 2k + 1$ and $e_f(2) = c(3k+1) + 2k$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$.

Case iii. $n \equiv 1 \pmod{3}$ and let $n = 3k + 1$. We consider the following subcases:

Subcase i. $m = 3c$.

Define f by $f(u_i) = 1$ for $1 \leq i \leq m$ and

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \leq j \leq c(3k+2) \\ 2 & \text{for } c(3k+2) < j \leq 2c(3k+2) \\ 1 & \text{for } j > 2c(3k+2). \end{cases}$$

Hence we have, $v_f(0) = v_f(1) = v_f(2) = c(3k+2)$ and $e_f(0) = e_f(1) = e_f(2) = c(3k+2)$. Therefore, $|e_f(i) - e_f(j)| = 0$ and $|v_f(i) - v_f(j)| = 0$ for all $i, j = 0, 1, 2$.

Subcase ii. $m = 3c + 1$.

Define f by $f(u_i) = 1$ for $1 \leq i \leq m$ and

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \leq j \leq c(3k+2) + k + 1 \\ 2 & \text{for } c(3k+2) + k + 1 < j \leq 2c(3k+2) + 2k + 1 \\ 1 & \text{for } j > 2c(3k+2) + 2k + 1. \end{cases}$$

Hence we have, $v_f(0) = v_f(1) = c(3k+2) + k + 1$, $v_f(2) = c(3k+2) + k$, $e_f(0) = e_f(1) = c(3k+2) + k + 1$ and $e_f(2) = c(3k+2) + k$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$.

Subcase iii. $m = 3c + 2$.

Define f by $f(u_i) = 1$ for $1 \leq i \leq m$ and

$$f(v_j) = \begin{cases} 0 & \text{for } 1 \leq j \leq c(3k+2) + 2k + 1 \\ 2 & \text{for } c(3k+2) + 2k + 1 < j \leq 2c(3k+2) + 4k + 2 \\ 1 & \text{for } j > 2c(3k+2) + 4k + 2. \end{cases}$$

Hence, $v_f(0) = v_f(2) = c(3k+2) + 2k + 1$, $v_f(1) = c(3k+2) + 2k + 2$, $e_f(0) = e_f(2) = c(3k+2) + 2k + 1$ and $e_f(1) = c(3k+2) + 2k + 2$. Therefore, $|e_f(i) - e_f(j)| \leq 1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Thus f is a 3-product cordial labeling. \square

If we delete an edge from a cycle C_m of $C_m \circ \overline{K_n}$, we get a graph $P_m \circ \overline{K_n}$. Hence, we have the following corollary.

Corollary 3.4. $P_m \circ \overline{K_n}$ is a 3-product cordial graph for $m, n \geq 1$.

Theorem 3.5. Fl_n is a 3-product cordial.

Proof. Let H_n be a helm with v as the apex vertex, v_1, v_2, \dots, v_n be the vertices of cycle and u_1, u_2, \dots, u_n be the pendant vertices for $n \geq 3$. Let Fl_n be the flower graph obtained from helm H_n . Then $|V(Fl_n)| = 2n + 1$ and $|E(Fl_n)| = 4n$. We define $f : V(Fl_n) \rightarrow \{0, 1, 2\}$ as follows:

Case (i) $n \equiv 0(mod 3)$.

$$f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq \frac{2n}{3} \\ 2 & \text{for } i = \frac{2n}{3} + j \text{ where } j \equiv 1, 3(mod 4), 1 \leq j < \frac{n}{3} \\ 1 & \text{for } i = \frac{2n}{3} + j \text{ where } j \equiv 0, 2(mod 4), 1 \leq j < \frac{n}{3} \end{cases}$$

$$f(u_n) = \begin{cases} 1 & \text{for } n \equiv 0(mod 4) \\ 2 & \text{Otherwise} \end{cases}$$

For $1 \leq i \leq n$, $f(v_i) = \begin{cases} 1 & \text{if } i \equiv 1, 2(mod 4) \\ 2 & \text{if } i \equiv 0, 3(mod 4) \end{cases}$

$$f(c) = 1.$$

We have $v_f(0) = v_f(1) - 1 = v_f(2) = \lfloor \frac{2n+1}{3} \rfloor$ and $e_f(0) = e_f(1) = e_f(2) = \frac{4n}{3}$.

Case (ii) $n \equiv 1(mod 3)$.

For $1 \leq i \leq n$, $f(v_i) = \begin{cases} 1 & \text{if } i \equiv 1, 2(mod 4) \\ 2 & \text{if } i \equiv 0, 3(mod 4) \end{cases}$, $f(c) = \begin{cases} 1 & \text{if } n \text{ is even} \\ 2 & \text{if } n \text{ is odd.} \end{cases}$

If n is even, $f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq \frac{2n+1}{3} \\ 2 & \text{for } i = \frac{2n+1}{3} + j \text{ where } j \equiv 1, 2(mod 4), 1 \leq j < \frac{n-1}{3} \\ 1 & \text{for } i = \frac{2n+1}{3} + j \text{ where } j \equiv 0, 3(mod 4), 1 \leq j < \frac{n-1}{3}. \end{cases}$

$$f(u_n) = 2.$$

If n is odd, $f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq \frac{2n+1}{3} \\ 1 & \text{for } i = \frac{2n+1}{3} + j \text{ where } j \equiv 1, 3(mod 4), 1 \leq j \leq \frac{n-1}{3} \\ 2 & \text{for } i = \frac{2n+1}{3} + j \text{ where } j \equiv 0, 2(mod 4), 1 \leq j \leq \frac{n-1}{3}. \end{cases}$

We have $v_f(0) = v_f(1) = v_f(2) = \frac{2n+1}{3}$ and $e_f(0) - 1 = e_f(1) = e_f(2) = \lfloor \frac{4n}{3} \rfloor$.

Case (iii) $n \equiv 2(mod 3)$.

$$f(c) = \begin{cases} 1 & \text{if } n \text{ is odd} \\ 2 & \text{if } n \text{ is even.} \end{cases}$$

$$f(u_i) = \begin{cases} 0 & \text{for } 1 \leq i \leq \lfloor \frac{2n}{3} \rfloor \\ 1 & \text{for } i = \lfloor \frac{2n}{3} \rfloor + j \text{ where } j \equiv 1, 2(mod 4), 1 \leq j \leq \frac{n}{3} \\ 2 & \text{for } i = \lfloor \frac{2n}{3} \rfloor + j \text{ where } j \equiv 0, 3(mod 4), 1 \leq j \leq \frac{n}{3}. \end{cases}$$

$$f(u_n) = 2.$$

$$\text{For } 1 \leq i \leq n, \text{ if } n \text{ is even, } f(v_i) = \begin{cases} 1 & \text{if } i \equiv 1, 2(mod 4) \\ 2 & \text{if } i \equiv 0, 3(mod 4). \end{cases}$$

$$\text{For } 1 \leq i \leq n-2, \text{ if } n \text{ is odd, } f(v_i) = \begin{cases} 1 & \text{if } i \equiv 1, 2(mod 4) \\ 2 & \text{if } i \equiv 0, 3(mod 4). \end{cases}$$

$$f(v_{n-1}) = f(v_n) = 2.$$

We have $v_f(0) + 1 = v_f(1) = v_f(2) = \lceil \frac{2n+1}{3} \rceil$ and $e_f(0) + 1 = e_f(1) = e_f(2) = \lceil \frac{4n}{3} \rceil$.

Thus in each case we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$.

Hence the graph Fl_n is a 3-product cordial graph. \square

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