SUT Journal of Mathematics
Vol. 49, No. 2 (2013), 109–119

Some transformations on Kenmotsu manifolds

Absos Ali Shaikh, Falleh R. Al-Solamy and Helaluddin Ahmad

(Received October 25, 2013; Revised January 11, 2014)

Abstract. The present paper deals with a study of certain transformations on a Kenmotsu manifold. We study an infinitesimal CL-transformation on a Kenmotsu manifold. We also study CL-transformation on a Kenmotsu manifold and obtain a new tensor field which is invariant under such a transformation. Finally we study CL-semisymmetric Kenmotsu manifold and prove that it is a manifold of constant curvature -1, from which we obtain some equivalent conditions as characterization of such a manifold.

AMS 2010 *Mathematics Subject Classification.* 53C15, 53C25, 53C35.

Key words and phrases. Kenmotsu manifold, infinitesimal CL-transformation, CL-transformation, CL-curvature, Einstein manifold, constant curvature, CL-flat, CL-symmetric, CL-semisymmetric.

§1. Introduction

In 1963, Tashiro and Tachibana [8] introduced a transformation, called CL-transformation, on a Sasakian manifold under which C-loxodrome remains invariant. We mention that a loxodrome is a curve on the unit sphere that intersects the meridians at a fixed angle, and the loxodrome was mainly used in navigation and usually called rhumb lines. We note that a C-loxodrome is a loxodrome cutting geodesic trajectories of the characterstic vector field ξ of the Sasakian manifold with constant angle. We also note that under conformal transformation angle between two intersecting curves remains invariant and the conformal curvature tensor is the invariant of such a transformation [9]. On the other hand, under CL-transformation the angle between two C-loxodromes remains invariant and hence 'CL' stands for C-loxodrome. The transformation was mainly defined for Sasakian manifold by Tashiro and Tachibana [8] and hence the invariance of such a transformation depends on specific manifold as the invariant tensor field on Sasakian manifold was determined by Koto and Nagao [3]. Also, an invariant tensor field on LP-Sasakian manifold was obtained by Matsumoto and Mihai [4]. Recently, the invariant tensor field under such a transformation on Lorentzian concircular structure manifold is investigated by Shaikh and Ahmad [6]. In the present paper the invariant tensor field under such a transformation on Kenmotsu manifold is obtained in Section 4. Again, Takamatsu and Mizusawa [7] studied an infinitesimal *CL*-transformation in a compact Sasakian manifold. We note that an infinitesimal *CL*-transformation means the vector field associated with a local one-parameter transformation group consisting of *CL*-transformations and may be named as a *CL-Killing vector field*.

In 1972, Kenmotsu [2] introduced a class of almost contact Riemannian manifolds which is called Kenmotsu manifold. We note that the structure of a Kenmotsu manifold is normal but not quasi-Sasakian and hence not Sasakian. We also note that a Kenmotsu manifold is not compact. Again, if F is a Kählerian manifold and c is a non-zero constant such that $g(t) = ce^t$ is a function on a line L, then the warped product $M = L \times_g F$ is a Kenmotsu manifold and the converse also is true [2].

The object of the present paper is to study some transformations on a Kenmotsu manifold. The paper is organized as follows. Section 2 provides the rudimentary facts of Kenmotsu manifolds along with some curvature relations. Section 3 is devoted to the study of an infinitesimal CL-transformation on a Kenmotsu manifold and it is proved that such a transformation is not necessarily a projective Killing vector field. However, on an Einstein Kenmotsu manifold an infinitesimal CL-transformation is necessarily a projective Killing vector field. In Section 4 we study a CL-transformation on a Kenmotsu manifold and obtain a new tensor field which is invariant under the CL-transformation. This tensor field is called a *CL-curvature tensor field*. In the last section we study CL-semisymmetric Kenmotsu manifolds and prove that such a manifold is of constant curvature -1 and the converse also is true. Hence in a Kenmotsu manifold, the concept of CL-semisymmetry, CL-symmetry, CL-flatness, semisymmetry, local symmetry, conformally flatness and manifold of constant curvature -1 are equivalent (see Corollary 5.1).

§2. Kenmotsu manifolds

A $(2n + 1)$-dimensional smooth manifold M is said to be an almost contact metric manifold [11] if there exist an (1,1) tensor field ϕ, a vector field ξ, an 1-form η and a Riemannian metric g on M such that

$$(2.1) \qquad \text{(a)}\ \eta(\xi) = 1, \qquad \text{(b)}\ \phi \circ \xi = 0, \qquad \text{(c)}\ \eta \circ \phi = 0,$$

$$(2.2) \qquad \text{(a)}\ \eta(X) = g(X, \xi), \qquad \text{(b)}\ \phi^2 X = -X + \eta(X)\xi,$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.3}$$

for any vector fields X and Y on M. An almost contact metric manifold M equipped with an almost contact metric structure (ϕ, ξ, η, g) is said to be a Kenmotsu manifold [2] if

$$(\nabla_X \phi)(Y) = -\eta(Y)\phi X - g(X, \phi Y)\xi \tag{2.4}$$

and

$$\nabla_X \xi = X - \eta(X)\xi \tag{2.5}$$

for any vector fields X and Y on M, where ∇ is the Levi-Civita connection of g.

In a Kenmotsu manifold, the following relations hold [2]:

$$(\nabla_X \eta)(Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.6}$$

$$\eta(R(X, Y)Z) = \{g(X, Z)\eta(Y) - g(Y, Z)\eta(X)\}, \tag{2.7}$$

$$R(\xi, Y)Z = \{\eta(Z)Y - g(Y, Z)\xi\}, \tag{2.8}$$

$$S(X, \xi) = -2n\eta(X), \tag{2.9}$$

$$(\nabla_Z R)(X, Y)\xi = -R(X, Y)Z - \{g(Y, Z)X - g(X, Z)Y\} \tag{2.10}$$

for any vector fields X, Y and Z on M, where R and S are the curvature tensor and the Ricci tensor of g respectively. For various results of Kenmotsu manifolds, we refer the reader to the book of Pitiş [5] and also references therein.

Throughout the paper we will consider a Kenmotsu manifold M of dimension $2n+1$, $n \geq 1$, endowed with the Levi-Civita connection ∇. In particular, from Section 4 to the last section we assume that $n > 1$.

§3. Infinitesimal CL-transformation on a Kenmotsu manifold

Definition 3.1. *A vector field V on a Kenmotsu manifold M is said to be an infinitesimal CL-transformation [7] if it satisfies*

$$\pounds_V \left\{ {h \atop ji} \right\} = \rho_j \delta_i^h + \rho_i \delta_j^h + \alpha(\eta_j \phi_i^h + \eta_i \phi_j^h) \tag{3.1}$$

for a certain constant α, where ρ_i are the components of an 1-form ρ, $\pounds_V$ denotes the Lie derivative with respect to V and $\left\{ {h \atop ji} \right\}$ is the Christoffel symbol of the Riemannian metric g.

Proposition 3.1. *If V is an infinitesimal CL-transformation on a Kenmotsu manifold, then the 1-form ρ is closed.*

Proof. Contracting h and j in (3.1), it can be easily seen that ρ_i is a gradient. Hence the 1-form ρ is closed. □

Theorem 3.1. *If V is an infinitesimal CL-transformation on a Kenmotsu manifold M, then the relation*

$$(\pounds_V g)(Y,Z) = (\nabla_Y \rho)(Z) - \alpha g(Y, \phi Z) \tag{3.2}$$

holds for any vector fields Y and Z on M.

Proof. It is known from [10] that

$$\pounds_V R^h_{kji} = \nabla_k \pounds_V \{^{\,h}_{ji}\} - \nabla_j \pounds_V \{^{\,h}_{ki}\}. \tag{3.3}$$

Substituting (3.1) into (3.3) and then using (2.4) and (2.6), we obtain

$$\begin{aligned}(\pounds_V R)(X,Y)Z = {} & (\nabla_X \rho)(Z)Y - (\nabla_Y \rho)(Z)X \\ & + \alpha[\{g(X,Z)\phi Y - g(Y,Z)\phi X\} \\ & - \{\eta(Y)\phi X - \eta(X)\phi Y\}\eta(Z) + \{g(Y,\phi Z)\eta(X) \\ & - g(X,\phi Z)\eta(Y) + 2g(Y,\phi X)\eta(Z)\}\xi]\end{aligned} \tag{3.4}$$

for any vector fields X, Y and Z on M. Operating η to (3.4), we get

$$\begin{aligned}\eta((\pounds_V R)(X,Y)Z) = {} & (\nabla_X \rho)(Z)\eta(Y) - (\nabla_Y \rho)(Z)\eta(X) \\ & + \alpha\{g(Y,\phi Z)\eta(X) - g(X,\phi Z)\eta(Y) \\ & + 2g(Y,\phi X)\eta(Z)\}.\end{aligned} \tag{3.5}$$

Taking Lie derivative of (2.7) with respect to V and using (3.5) and then replacing X and Y to Y and ξ, respectively, we get

$$(\pounds_V g)(Y,Z) = (\nabla_Y \rho)(Z) - \{(\nabla_\xi \rho)(Z) - (\pounds_V g)(\xi,Z)\}\eta(Y) - \alpha g(Y,\phi Z). \tag{3.6}$$

Interchanging Y and Z in (3.6) and then subtracting it from (3.6), we get

$$\begin{aligned}\{(\nabla_\xi \rho)(Z) - (\pounds_V g)(\xi,Z)\}\eta(Y) = {} & \{(\nabla_\xi \rho)(Y) - (\pounds_V g)(\xi,Y)\}\eta(Z) \\ & + 2\alpha g(\phi Y, Z).\end{aligned} \tag{3.7}$$

Replacing Y to ξ in (3.7) we obtain

$$(\nabla_\xi \rho)(Z) - (\pounds_V g)(\xi,Z) = \{(\nabla_\xi \rho)(\xi) - (\pounds_V g)(\xi,\xi)\}\eta(Z). \tag{3.8}$$

From (3.6) and (3.8), we obtain

$$(\pounds_V g)(Y,Z) = (\nabla_Y \rho)(Z) - \{(\nabla_\xi \rho)(\xi) - (\pounds_V g)(\xi,\xi)\}\eta(Y)\eta(Z) - \alpha g(Y,\phi Z). \tag{3.9}$$

Now taking inner product of (3.4) with a vector field W on M and then contracting X and W, we get

$$(\pounds_V S)(Y,Z) = -2n(\nabla_Y \rho)(Z). \tag{3.10}$$

Replacing Y to ξ in (3.10), we have

$$(\pounds_V S)(\xi,Z) = -2n(\nabla_\xi \rho)(Z). \tag{3.11}$$

Taking Lie derivative of (2.9) with respect to V and using (3.11) and then replacing Z to ξ, we obtain

$$(\nabla_\xi \rho)(\xi) = (\pounds_V g)(\xi,\xi). \tag{3.12}$$

Using (3.12) in (3.9), we obtain (3.2). This completes the proof. □

From (3.2), we can state the following:

Theorem 3.2. *An infinitesimal CL-transformation V on a Kenmotsu manifold M is not a projective Killing vector field unless $\alpha = 0$.*

Corollary 3.1. *Any infinitesimal CL-transformation V on an Einstein Kenmotsu manifold M is necessarily a projective Killing vector field.*

Proof. Let M be an Einstein Kenmotsu manifold. Then

$$S(Y,Z) = -2ng(Y,Z). \tag{3.13}$$

Taking Lie derivative of (3.13) with respect to V and using (3.2) and (3.10), we obtain

$$\alpha g(Y,\phi Z) = 0 \tag{3.14}$$

from which we get $\alpha = 0$. This completes the proof. □

Corollary 3.2. *If V is an infinitesimal CL-transformation on an Einstein Kenmotsu manifold M, then $V - \frac{1}{2}\mu$ is a Killing vector field, where μ is the associated vector field of the 1-form ρ.*

§4. CL-transformation on a Kenmotsu manifold

Definition 4.1 ([3]). *A transformation f on a $(2n+1)$-dimensional Kenmotsu manifold M with structure (ϕ, ξ, η, g) is said to be a CL-transformation if the Levi-Civita connection ∇ and a symmetric affine connection ∇^f induced from ∇ by f are related by*

$$\nabla^f_X Y = \nabla_X Y + \rho(X)Y + \rho(Y)X + \alpha\{\eta(X)\phi Y + \eta(Y)\phi X\}, \tag{4.1}$$

where ρ is an 1-form and α is a constant.

Throughout the section, the geometric objects with respect to the symmetric affine connection ∇^f are represented as R^f and S^f etc., where R^f and S^f denote the curvature tensor and the Ricci tensor of the connection ∇^f respectively.

If f is a CL-transformation on a Kenmotsu manifold M, then by virtue of (4.1), (2.2), (2.4) and (2.6), the curvature tensor $R^f(X,Y)Z$ of the connection ∇^f is given by

$$\begin{aligned} R^f(X,Y)Z = R(X,Y)Z &+ \{B(X,Y) - B(Y,X)\}Z + B(X,Z)Y \\ &- B(Y,Z)X - \alpha[\{\eta(Y)\phi X - \eta(X)\phi Y\}\eta(Z) \\ &+ \{g(Y,Z)\phi X - g(X,Z)\phi Y\} - \{g(Y,\phi Z)\eta(X) \\ &- g(X,\phi Z)\eta(Y) - 2g(X,\phi Y)\eta(Z)\}\xi], \end{aligned} \tag{4.2}$$

for any vector fields X, Y and Z on M, where the tensor field $B(X,Y)$ is defined by

$$\begin{aligned} B(X,Y) = (\nabla_X\rho)(Y) - \rho(X)\rho(Y) - \alpha^2\eta(X)\eta(Y) \\ - \alpha\{\eta(X)\rho(\phi Y) + \eta(Y)\rho(\phi X)\}. \end{aligned} \tag{4.3}$$

From (4.2), we have

$$\begin{aligned} g(R^f(X,Y)Z,U) = g(R(X,Y)Z,U) &+ \{B(X,Y) - B(Y,X)\}g(Z,U) \\ &+ B(X,Z)g(Y,U) - B(Y,Z)g(X,U) \\ &- \alpha\big[\{\eta(Y)g(\phi X,U) - \eta(X)g(\phi Y,U)\}\eta(Z) \\ &+ \{g(Y,Z)g(\phi X,U) - g(X,Z)g(\phi Y,U)\} \\ &- \{g(Y,\phi Z)\eta(X) - g(X,\phi Z)\eta(Y) \\ &- 2g(X,\phi Y)\eta(Z)\}\eta(U)\big], \end{aligned} \tag{4.4}$$

where U is any vector field on M. Taking contraction of (4.4) over X and U and also over Z and U and then adding the results and proceeding same manner as in [3], it is easy to check that $B(X,Y)$ is symmetric and hence from (4.3) we see that the 1-form ρ is closed.

Theorem 4.1. *Let A be the tensor field of type $(1,3)$ given by*

$$(4.5)\quad \begin{aligned} A(X,Y)Z := R(X,Y)Z - \frac{1}{2n}\big[&\{S(Y,Z)X - S(X,Z)Y\} \\ &- \{g(Y,Z) + \eta(Y)\eta(Z)\}QX + \{g(X,Z) + \eta(X)\eta(Z)\}QY \\ &+ [\{S(X,Z) + 2ng(X,Z)\}\eta(Y) - \{S(Y,Z) \\ &+ 2ng(Y,Z)\}\eta(X) + 2\{S(X,Y) + 2ng(X,Y)\}\eta(Z)]\xi\big] \\ &+ \{g(Y,Z) + \eta(Y)\eta(Z)\}X - \{g(X,Z) + \eta(X)\eta(Z)\}Y, \end{aligned}$$

where Q is the tensor field of type $(1,1)$ given by $g(QX,Y) = S(X,Y)$, where X and Y are vector fields on M. This tensor field A is invariant under CL-transformations of a Kenmotsu manifold M.

Proof. Assume that f is a CL-transformation on a Kenmotsu manifold M. Then the relations (4.1)-(4.4) hold. Since the tensor $B(X,Y)$ is symmetric, (4.2) can be written as

$$(4.6)\quad \begin{aligned} R^f(X,Y)Z = R(X,Y)Z + B(X,Z)Y - B(Y,Z)X - \alpha[&\{\eta(Y)\phi X \\ &- \eta(X)\phi Y\}\eta(Z) + \{g(Y,Z)\phi X - g(X,Z)\phi Y\} \\ &- \{g(Y,\phi Z)\eta(X) - g(X,\phi Z)\eta(Y) - 2g(X,\phi Y)\eta(Z)\}\xi], \end{aligned}$$

which yields

$$(4.7)\qquad 2nB(Y,Z) = S(Y,Z) - S^f(Y,Z).$$

Substituting (4.7) into (4.6), we obtain

$$(4.8)\qquad \begin{aligned} P^f(X,Y)Z = P(X,Y)Z - \alpha[&\{\eta(Y)\phi X - \eta(X)\phi Y\}\eta(Z) \\ &+ \{g(Y,Z)\phi X - g(X,Z)\phi Y\} - \{g(Y,\phi Z)\eta(X) \\ &- g(X,\phi Z)\eta(Y) - 2g(X,\phi Y)\eta(Z)\}\xi], \end{aligned}$$

where P is the projective curvature tensor [9] given by

$$(4.9)\qquad P(X,Y)Z = R(X,Y)Z - \frac{1}{2n}\{S(Y,Z)X - S(X,Z)Y\}.$$

Replacing X to ξ in (4.8) and operating η, we obtain

$$(4.10)\qquad \eta(P^f(\xi,Y)Z) - \eta(P(\xi,Y)Z) = \alpha g(Y,\phi Z).$$

From (4.8) and (4.10), we have

$$(4.11)\qquad \begin{aligned} H^f(X,Y)Z = H(X,Y)Z - \alpha[&\{g(Y,Z)\phi X - g(X,Z)\phi Y\} \\ &+ \{\eta(Y)\phi X - \eta(X)\phi Y\}\eta(Z)], \end{aligned}$$

where we put

$$H(X,Y)Z = P(X,Y)Z - \{\eta(P(\xi,Y)Z)\eta(X) - \eta(P(\xi,X)Z)\eta(Y) - 2\eta(P(\xi,X)Y)\eta(Z)\}\xi. \tag{4.12}$$

Taking the inner product of (4.11) with W and then using (4.10), we obtain

$$g(L^f(X,Y)Z,W) = g(L(X,Y)Z,W) \tag{4.13}$$

where the tensor field L is defined by

$$\begin{aligned} g(L(X,Y)Z,W) = g(H(X,Y)Z,W) &\\ + \{g(X,Z) + \eta(X)\eta(Z)\}\eta(P(\xi,Y)W) &\\ - \{g(Y,Z) + \eta(Y)\eta(Z)\}\eta(P(\xi,X)W) & \end{aligned} \tag{4.14}$$

and L^f also is defined similarly. Using (4.9), (4.12), (2.7), (2.9) and (4.14), we obtain

$$g(L(X,Y)Z,W) = g(A(X,Y)Z,W),$$

that is, $L = A$. Similarly we obtain $L^f = A^f$. Hence we obtain

$$g(A^f(X,Y)Z,W) = g(A(X,Y)Z,W).$$

This completes the proof. □

This tensor field A on a Kenmotsu manifold M invariant under a CL-transformation is said to be the CL-curvature tensor field on M.

§5. CL-semisymmetric Kenmotsu manifolds

Definition 5.1. *A Kenmotsu manifold M is said to be CL-flat if the CL-curvature tensor field A of type (1,3) vanishes identically on M.*

We mention that CL-flat manifold was introduced by Koto and Nagao in [3] for a Sasakian manifold.

Definition 5.2. *A Kenmotsu manifold M is said to be CL-symmetric if $\nabla A = 0$.*

A Riemannian manifold M is said to be locally symmetric due to Cartan [1] if it satisfies $\nabla R = 0$.

Definition 5.3. *A Kenmotsu manifold M is said to be CL-semisymmetric if $R(U,W) \cdot A = 0$.*

From (4.5), we have

$$
\begin{aligned}
(5.1)\qquad & 2n(R(U,W)\cdot A)(X,Y)Z \\
&= 2n(R(U,W)\cdot R)(X,Y)Z \\
&\quad - \{(R(U,W)\cdot S)(Y,Z)X - (R(U,W)\cdot S)(X,Z)Y\} \\
&\quad + [\{g(Y,Z)+\eta(Y)\eta(Z)\}(R(U,W)\cdot Q)(X) \\
&\quad - \{g(X,Z)+\eta(X)\eta(Z)\}(R(U,W)\cdot Q)(Y)] \\
&\quad - \{(R(U,W)\cdot S)(X,Z)\eta(Y) - (R(U,W)\cdot S)(Y,Z)\eta(X) \\
&\quad + 2(R(U,W)\cdot S)(X,Y)\eta(Z)\}\xi - \{E(X,Z)\eta(Y) \\
&\quad - E(Y,Z)\eta(X) + 2E(X,Y)\eta(Z)\}\{\eta(U)W - \eta(W)U\} \\
&\quad + \{g(U,Y)\eta(W) - g(W,Y)\eta(U)\}\{E(X,Z)\xi - \eta(Z)\mathcal{E}X\} \\
&\quad - \{g(U,X)\eta(W) - g(W,X)\eta(U)\}\{E(Y,Z)\xi - \eta(Z)\mathcal{E}Y\} \\
&\quad + \{g(U,Z)\eta(W) - g(W,Z)\eta(U)\}[2E(X,Y)\xi \\
&\quad - \eta(Z)\{\eta(Y)\mathcal{E}X - \eta(X)\mathcal{E}Y\}],
\end{aligned}
$$

where we put

$$
(5.2)\qquad E(X,Y) = S(X,Y) + 2ng(X,Y)
$$

and $\mathcal{E}$ is given by $g(\mathcal{E}X,Y) = E(X,Y)$.

Lemma 5.1. *A Kenmotsu manifold M is CL-semisymmetric if and only if*

$$
\begin{aligned}
(5.3)\qquad & 2n(R(U,W)\cdot R)(X,Y)Z \\
&= \{(R(U,W)\cdot S)(Y,Z)X - (R(U,W)\cdot S)(X,Z)Y\} \\
&\quad - [\{g(Y,Z)+\eta(Y)\eta(Z)\}(R(U,W)\cdot Q)(X) \\
&\quad - \{g(X,Z)+\eta(X)\eta(Z)\}(R(U,W)\cdot Q)(Y)] \\
&\quad + \{(R(U,W)\cdot S)(X,Z)\eta(Y) - (R(U,W)\cdot S)(Y,Z)\eta(X) \\
&\quad + 2(R(U,W)\cdot S)(X,Y)\eta(Z)\}\xi + \{E(X,Z)\eta(Y) \\
&\quad - E(Y,Z)\eta(X) + 2E(X,Y)\eta(Z)\}\{\eta(U)W - \eta(W)U\} \\
&\quad - \{g(U,Y)\eta(W) - g(W,Y)\eta(U)\}\{E(X,Z)\xi - \eta(Z)\mathcal{E}X\} \\
&\quad + \{g(U,X)\eta(W) - g(W,X)\eta(U)\}\{E(Y,Z)\xi - \eta(Z)\mathcal{E}Y\} \\
&\quad - \{g(U,Z)\eta(W) - g(W,Z)\eta(U)\}[2E(X,Y)\xi \\
&\quad - \eta(Z)\{\eta(Y)\mathcal{E}X - \eta(X)\mathcal{E}Y\}]
\end{aligned}
$$

for any vector fields X, Y, Z, U and W on M.

Proof. The result follows from (5.1). □

Lemma 5.2. *A CL-semisymmetric Kenmotsu manifold M is Einstein and Ricci semisymmetric.*

Proof. Let M be a CL-semisymmetric Kenmotsu manifold. Then according to Lemma 5.1 we have the relation (5.3). Replacing X and Z to ξ in (5.3), we obtain

$$(R(U,W)\cdot Q)(Y) = \{E(W,Y)\eta(U) - E(U,Y)\eta(W)\}\xi. \tag{5.4}$$

Again, replacing U to ξ in (5.4) and operating η, we get

$$S(W,Y) = -2ng(W,Y), \tag{5.5}$$

i.e., M is an Einstein manifold. Now in view of (5.5), (5.4) yields

$$(R(U,W)\cdot Q)(Y) = 0. \tag{5.6}$$

Thus the manifold M is Ricci semisymmetric. □

Remark 5.1. *A CL-symmetric (resp. CL-flat) Kenmotsu manifold M is Einstein and Ricci symmetric.*

Theorem 5.1. *A Kenmotsu manifold M is CL-semisymmetric if and only if it is a manifold of constant curvature -1.*

Proof. Assume that M is CL-semisymmetric. Then replacing X and W to ξ in (5.3) and using (5.5), we obtain

$$R(U,Y)Z = -\{g(Y,Z)U - g(U,Z)Y\}, \tag{5.7}$$

i.e. M is of constant curvature -1.

Conversely if M is of constant curvature -1, then (5.7) implies (5.5) and hence it follows from (5.1) that M is CL-semisymmetric. □

From Theorem 5.1, we can state the following:

Corollary 5.1. *In a Kenmotsu manifold M, the following assertions are equivalent:*
(a) *M is CL-semisymmetric;*
(b) *M is CL-symmetic;*
(c) *M is CL-flat;*
(d) *M is semisymmetric;*
(e) *M is locally symmetric;*
(f) *M is conformally flat;*
(g) *M is a manifold of constant curvature -1.*

Acknowledgement: The authors wish to express their sincere thanks to the referee for his valuable comments towards the improvement of the paper.

References

[1] E. Cartan, *Sur une classe remarquable d'espaces de Riemannian*, Bull. Soc. Math. France, **54** (1926), 214-264.

[2] K. Kenmotsu, *A class of almost contact Riemannian manifolds*, Tôhoku Math. J., **24** (1972), 93-103.

[3] S. Koto and M. Nagao, *On an invariant tensor under a CL-transformation*, Kōdai Math. Sem. Rep., **18** (1966), 87-95.

[4] K. Matsumoto and I. Mihai, *On a certain transformation in a Lorentzian para-Sasakain manifold*, Tensor (N. S.), **47** (1988), 189-197.

[5] G. Pitiş, *Geometry of Kenmotsu manifolds*, Transilvania University Press, Braşov, **2007**.

[6] A. A. Shaikh and H. Ahmad, *Some transformations on $(LCS)_n$-manifolds*, to appear in Tsukuba J. Math., 2014.

[7] K. Takamatsu and H. Mizusawa, *On infinitesimal CL-transformations of compact normal contact metric spaces*, Sci. Rep. Niigata Univ., Ser. A, **3** (1966), 31-40.

[8] Y. Tashiro and S. Tachibana, *On Fubinian and C-Fubinian manifold*, Kōdai Math. Sem. Rep., **15** (1963), 176-183.

[9] H. Weyl, *Reine infinitesimal geometrie*, Math. Zeitschrift, **2** (1918), 384-411.

[10] K. Yano, *Differential geometry on complex and almost complex spaces*, Pergamon Press, **1965**.

[11] K. Yano and M. Kon, *Structures on manifolds*, World Scientific Publ., Singapore, **1984**.

Absos Ali Shaikh and Helaluddin Ahmad
Department of Mathematics,
University of Burdwan,
Burdwan – 713 104,
West Bengal, India.
E-mail: aask2003@yahoo.co.in, aashaikh@math.buruniv.ac.in

F. R. Al-Solamy
Department of Mathematics,
Faculty of Science, King Abdulaziz University,
Jeddah 21589,
Saudi Arabia.,
E-mail: falleh@hotmail.com