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α-almost Ricci solitons on Kenmotsu manifolds

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Abstract. The current article purports to investigate α-almost Ricci solitons in the framework of Kenmotsu manifolds. Among others, we prove that an α-almost Ricci solitons on a Kenmotsu manifold is expanding. Furthermore, we take into account α-almost Ricci solitons on Kenmotsu manifolds with Codazzi type of Ricci tensor and cyclic parallel Ricci tensor. Also, Kenmotsu manifolds satisfying the curvature condition $R.R = Q(S, R)$ is studied. Ultimately, we consider an example to prove the non-existence of proper α-almost Ricci solitons on Kenmotsu manifolds and verify some results.

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§1. Introduction

The methods of contact geometry performed a crucial role in present-day mathematics, and therefore it became famous among the eminent researchers. Contact geometry has manifested from the mathematical formalism of classical mechanics. The essences of contact geometry lie in differential equations as in 1980, and Lie [16] publicized the concept of contact transformation as a geometric tool to study systems of differential equations. This subject has many connections with the other fields of pure mathematics. In the offering exposition, we are entering some generalizations which perform an outstanding role in coeval mathematics.

Several years ago, to find a canonical metric on a smooth manifold, Hamilton [13] revealed the idea of Ricci flow. A Ricci soliton is nothing but a generalization of an Einstein metric. We recollect the idea of Ricci soliton, according to [4]. Let us presume a manifold M endowed with a Riemannian metric g, a vector field V, called potential vector field and λ a real scalar such that

$$\pounds_V g + 2S + 2\lambda g = 0, \tag{1.1}$$

where $\pounds$ indicates the Lie derivative. Then the triple (g, V, λ) named as Ricci soliton. Metrics satisfying (1.1) are fascinating and useful in physics and are often mentioned as quasi-Einstein. The Ricci soliton is named as shrinking, steady, and expanding according to λ is negative, zero and positive, respectively. Ricci solitons have been studied by several authors such as ([13],[19]) and many others.

Ricci solitons have been generalized in several ways, such as almost Ricci solitons ([10],[17]), η-Ricci solitons ([1],[2]), generalized Ricci soliton, $*$-Ricci solitons and many others.

A few years ago, taking the constant λ as a smooth function, Pigola et al. [17] introduced a new idea named as almost Ricci soliton, which is a generalization of Ricci soliton. Recently, the concept of almost Ricci soliton to α-almost Ricci soliton (briefly, α-ARS) is extended by Gomes et al. [10] on a complete Riemannian manifold by

$$\frac{\alpha}{2}\pounds_V g + S + \lambda g = 0, \tag{1.2}$$

where $\alpha : M \to \mathbb{R}$ is a smooth function. In particular, a Ricci Soliton is the 1-ARS with constant λ. More recently in [9], Ghosh and Patra studied α-ARS on K-contact metric manifolds and furnish some fascinating results.

Motivated by the above studies, we make the contribution to investigate an α-ARS in the framework of Kenmotsu manifolds.

The current article is constructed as follows: In section 2, we recall a few fundamental facts and formulas of Kenmotsu manifolds which we will need throughout the paper. Then we prove that an α-almost Ricci solitons on a Kenmotsu manifold is expanding. Moreover, we consider α-almost Ricci solitons on Kenmotsu manifolds with Codazzi type of Ricci tensor and cyclic parallel Ricci tensor. In the next section, Kenmotsu manifolds satisfying the curvature condition $R.R = Q(S, R)$ are studied. Then we consider an example to verify the results of our paper. This paper terminates with a small bibliography which by no means is exhaustive but contains only those references which have been consulted during the preparation of the present paper.

§2. Preliminaries

Let M^{2n+1} be a connected almost contact metric manifold endowed with an almost contact metric structure (ϕ,ξ,η,g), that is, ϕ is an (1,1)-tensor field, ξ

is a vector field, η is a 1-form and g is a compatible Riemannian metric such that

$$\phi^2(U) = -U + \eta(U)\xi, \eta(\xi) = 1, \phi\xi = 0, \eta\phi = 0 \tag{2.1}$$

$$g(\phi U, \phi V) = g(U, V) - \eta(U)\eta(V) \tag{2.2}$$

$$g(U, \xi) = \eta(U) \tag{2.3}$$

for all $U, V \in \Gamma(TM)$.
If the following condition is fulfilled in an almost contact metric manifold

$$(\nabla_U \phi)V = g(\phi U, V)\xi - \eta(V)\phi U, \tag{2.4}$$

then M is called a Kenmotsu manifold [14], where ∇ denotes the Levi-Civita connection of g. From the antecedent equation it is clear that

$$\nabla_U \xi = U - \eta(U)\xi \tag{2.5}$$

and

$$(\nabla_U \eta)V = g(U, V) - \eta(U)\eta(V). \tag{2.6}$$

In addition, the curvature tensor R and the Ricci tensor S satisfy

$$R(U, V)\xi = \eta(U)V - \eta(V)U, \tag{2.7}$$

$$R(\xi, U)V = \eta(V)U - g(U, V)\xi, \tag{2.8}$$

$$R(\xi, U)\xi = U - \eta(U)\xi, \tag{2.9}$$

and

$$S(U, \xi) = -2n\eta(U). \tag{2.10}$$

Kenmotsu manifolds have been studied by several authors such as Pitis [18], De [5], De, Yildiz and Yaliniz [6] and many others.

§3. Kenmotsu manifolds admitting α-ARS

Let $(g, \xi, \alpha, \lambda)$ be an α-ARS on a Kenmotsu manifold M^{2n+1}. Then we have

$$\frac{\alpha}{2}(\pounds_\xi g)(U,V) + S(U,V) + \lambda g(U,V) = 0. \tag{3.1}$$

In a Kenmotsu manifold we infer

$$(\pounds_\xi g)(U,V) = 2[g(U,V) - \eta(U)\eta(V)]. \tag{3.2}$$

Substituting the value of $(\pounds_\xi g)(U,V)$ from (3.2) in (3.1) we lead

$$S(U,V) = -(\alpha+\lambda)g(U,V) + \alpha\eta(U)\eta(V). \tag{3.3}$$

Now superseding U and V by ξ in the foregoing equation and utilizing (2.10) we get

$$\lambda = 2n. \tag{3.4}$$

Thus we conclude the following:

Theorem 3.1. *If a Kenmotsu manifold M admits an α-ARS $(g, \xi, \alpha, \lambda)$, then the Ricci soliton is expanding.*

§4. α-ARS on Kenmotsu manifolds with Codazzi type of Ricci tensor

In this section we consider proper α-ARS on Kenmotsu manifolds with Codazzi type of Ricci tensor. The amusing idea of cyclic parallel Ricci tensor and Codazzi type of Ricci tensor was introduced by Gray [12]. Codazzi type of Ricci tensor means that the Levi-Civita connection ∇ of such metric is a Yang-Mills connection while keeping the metric of the manifold fixed.

Bourguigon [3] proved the interesting result that any metric with Codazzi type of Ricci tensor on a compact orientable 4-manifold with non-vanishing signature is Einstein.

If the non-zero Ricci tensor S of type (0,2) of a Riemannian manifold satisfies the condition

$$(\nabla_U S)(V,W) + (\nabla_V S)(W,U) + (\nabla_W S)(U,V) = 0, \tag{4.1}$$

then the manifold is said to satisfy cyclic parallel Ricci tensor. Analogously the manifold is said to have Codazzi type of Ricci tensor if it satisfies the following condition

$$(\nabla_U S)(V,W) = (\nabla_V S)(U,W). \tag{4.2}$$

Therefore, taking covariant differentiation of (3.3) with respect to W we obtain

$$\begin{aligned}(\nabla_W S)(U,V) &= (W\alpha)[\eta(U)\eta(V)-g(U,V)]-(W\lambda)g(U,V)\\ &+ \alpha[g(U,W)\eta(V)+g(V,W)\eta(U)\\ &- 2\eta(U)\eta(V)\eta(W)].\end{aligned} \tag{4.3}$$

If the Ricci tensor S is of Codazzi type, then

$$(\nabla_W S)(U,V) = (\nabla_U S)(W,V). \tag{4.4}$$

Using (4.3) in (4.4) we have

$$\begin{aligned}&(W\alpha)[\eta(U)\eta(V)-g(U,V)]-(W\lambda)g(U,V)\\ &+ \alpha[g(U,W)\eta(V)+g(V,W)\eta(U)-2\eta(U)\eta(V)\eta(W)]\\ &= (U\alpha)[\eta(U)\eta(V)-g(U,V)]-(W\lambda)g(U,V)\\ &+ \alpha[g(W,U)\eta(V)+g(V,W)\eta(U)-2\eta(W)\eta(V)\eta(U)].\end{aligned} \tag{4.5}$$

Putting $V=\xi$ in (4.5) and using (3.4), we lead

$$\alpha[g(U,W)-\eta(U)\eta(W)]=0. \tag{4.6}$$

If $g(U,W)-\eta(U)\eta(W)=0$, then we have $g(\phi U,\phi W)=0$. Pellucidly, operating ϕ and utilizing skew-symmetry property we can facilely obtain $\phi^2 U=0$, which is a contradiction. Thus we infer $\alpha=0$. Hence a Kenmotsu manifold with Codazzi type of Ricci tensor does not admit a proper α-ARS. Thus we conclude the following:

Theorem 4.1. *A Kenmotsu manifold with Codazzi type of Ricci tensor does not admit a proper α-ARS.*

Remark 4.1. *From the above Theorem using (3.3) we obtain that the manifold is an Einstein manifold.*

§5. α-ARS on Kenmotsu manifolds with cyclic parallel Ricci tensor

This section is dedicated to study a proper α-ARS on Kenmotsu manifolds with cyclic parallel Ricci tensor. Ki et al [15] proved that Carten hypersurfaces are manifold, with non-parallel Ricci tensor satisfies cyclic parallel Ricci tensor (that is, $\nabla_U S)(V,W)+(\nabla_V S)(W,U)+(\nabla_W S)(U,V)=0$). Therefore

$$(\nabla_U S)(V,W)+(\nabla_V S)(W,U)+(\nabla_W S)(U,V)=0, \tag{5.1}$$

for all smooth vector fields $U, V, W \in \Gamma(TM)$. Putting $U = \xi$ in (5.1) and using (4.3), and maintaining the same procedure as in the proof of Theorem 4.1, we can easily obtain

$$\alpha[g(V, W) - \eta(V)\eta(W)] = 0.$$

Then following the same reason as in the proof of Theorem4.1 we lead

$$\alpha = 0. \tag{5.2}$$

Thus we are in a position to state the following:

Theorem 5.1. *A Kenmotsu manifold with cyclic parallel Ricci tensor does not admit a proper α-ARS.*

§6. α-ARS on Kenmotsu manifolds satisfying $R.R = Q(S, R)$

In this section we discuss with α-ARS on Kenmotsu manifolds satisfying $R.R = Q(S, R)$, where Q is the Ricci operator defined by $g(QU, V) = S(U, V)$, S is the Ricci tensor of type $(0, 2)$. Linearly dependent condition on the tensors $R.R$ and $Q(S, R)$ implies that the manifold M is Ricci generalized pseudosymmetric. Which is equivalent to

$$R.R = fQ(S, R),$$

holding on the set $X_R = \{x \in M : R \neq 0 \text{ at } x\}$, where f is some function on X_R. A very useful subclass of this class of manifolds fixing the condition is

$$R.R = Q(S, R). \tag{6.1}$$

The manifolds fulfilling the condition $R.R = Q(S, R)$ were studied in [7]. Conformally flat manifolds realizing (6.1) were considered in [8] and also show that the above equation identically [8] satisfied by every 3-dimensional Riemannian manifold. Now from (6.1) we obtain

$$R.R = Q(S, R), \tag{6.2}$$

that is,

$$(R(U, V).R)(X, Y)Z = ((U \wedge_S V) \cdot R)(X, Y)Z. \tag{6.3}$$

We lead from (6.3)

$$\begin{aligned} &R(U, V)R(X, Y)Z - R(R(U, V)X, Y)Z \\ &-R(X, R(U, V)Y)Z - R(X, Y)R(U, V)Z \\ &= (X \wedge_S Y)R(U, V)W - R((X \wedge_S Y)U, V)W \\ &-R(X, (U \wedge_S V)Y)Z - R(X, Y)(U \wedge_S V)Z. \end{aligned} \tag{6.4}$$

We define endomorphisms $U \wedge_A V$ by

$$(U \wedge_A V)W = A(V,W)U - A(U,W)V, \tag{6.5}$$

where $U, V, W \in \Gamma(TM)$ and A is a symmetric $(0,2)$-tensor. In view of (6.5) and (6.4) we infer

$$\begin{aligned} &R(U,V)R(X,Y)Z - R(R(U,V)X,Y)Z - R(X,R(U,V)Y)Z \\ &-R(X,Y)R(U,V)Z = S(Y,R(U,V)W)X - S(U,R(X,Y)Z)V \\ &-S(V,X)R(U,Y)Z + S(U,X)R(V,Y)Z - S(V,Y)R(X,U)Z \\ &+S(U,Y)R(X,V)Z - S(V,Z)R(X,Y)U + S(U,Z)R(X,Y)V. \end{aligned} \tag{6.6}$$

Substituting $X = U = \xi$ in (6.6) and utilizing (2.7),(2.8),(2.9) and (2.10) we obtain

$$\begin{aligned} &-g(Y,Z)V + g(V,Z)Y - R(V,Y)Z \\ &= \eta(Z)S(V,Y)\xi - 2g(Y,Z)V - 2nR(V,Y)Z \\ &+2ng(V,Z)\eta(Y)\xi - S(V,Z)Y + S(V,Z)\eta(Y)\xi \\ &+2ng(V,Y)\eta(Z)\xi. \end{aligned} \tag{6.7}$$

Taking the inner product of (6.7) with W yields

$$\begin{aligned} &g(R(V,Y)Z,W) + g(Y,Z)g(V,W) + g(V,Z)g(Y,W) \\ &+S(V,Z)g(Y,W) - S(V,Y)\eta(W)\eta(Z) - S(V,Z)\eta(Y)\eta(W) \\ &-2ng(V,Y)\eta(W)\eta(Z) - 2ng(V,Z)\eta(Y)\eta(W) = 0. \end{aligned} \tag{6.8}$$

Let $\{e_i\}(1 \leq i \leq n)$ be an orthonormal basis of the tangent space at any point. Now taking summation over $i = 1, 2, 3, ...n$ of the relation (6.8) for $Y = Z = e_i$ gives

$$S(V,W) = -2ng(V,W). \tag{6.9}$$

Also, from (3.3) using (3.4) we lead

$$S(V,W) = -(\alpha + 2n)g(V,W) + \alpha\eta(V)\eta(W). \tag{6.10}$$

In view of (6.9) and (6.10) we infer

$$\alpha[g(V,W) - \eta(V)\eta(W)] = 0. \tag{6.11}$$

Then following the same argument as in the proof of Theorem4.1 we infer $\alpha = 0$, which leads to the following:

Theorem 6.1. *A Kenmotsu manifold with the curvature condition $R.R = Q(S,R)$ does not admit a proper α-ARS.*

§7. Example

Here we consider the example of the paper [11]. Let us consider the 5-dimensional manifold $M = \{(x, y, z, u, v) \in \mathbb{R}^5\}$, where (x, y, z, u, v) are the standard coordinates in $\mathbb{R}^5$.
We choose the vector fields
$\delta_1 = e^{-v}\frac{\partial}{\partial x}$, $\delta_2 = e^{-v}\frac{\partial}{\partial y}$, $\delta_3 = e^{-v}\frac{\partial}{\partial z}$, $\delta_4 = e^{-v}\frac{\partial}{\partial u}$, $\delta_5 = \frac{\partial}{\partial v}$,
which are linearly independent at each point of M.
Let g be the Riemannian metric defined by $g(\delta_i, \delta_j) = 0$, $i \neq j$, $i, j = 1, 2, 3, 4, 5$ and

$$g(\delta_1, \delta_1) = g(\delta_2, \delta_2) = g(\delta_3, \delta_3) = g(\delta_4, \delta_4) = g(\delta_5, \delta_5) = 1. \tag{7.1}$$

Let η be the 1-form defined by $\eta(Z) = g(Z, \delta_5)$, for any $Z \in \Gamma(TM)$, where $\Gamma(TM)$ is the set of all differentiable vector fields on M.
Let ϕ be the $(1, 1)$-tensor field defined by

$$\phi\delta_1 = \delta_3, \tag{7.2}$$

$$\phi\delta_2 = \delta_4, \tag{7.3}$$

$$\phi\delta_3 = -\delta_1, \tag{7.4}$$

$$\phi\delta_4 = -\delta_2, \tag{7.5}$$

$$\phi\delta_5 = 0. \tag{7.6}$$

Using the linearity of ϕ and g, we have

$$\eta(\delta_5) = 1, \tag{7.7}$$

$$\phi^2 Z = -Z + \eta(Z)\delta_5 \tag{7.8}$$

and

$$g(\phi Z, \phi U) = g(Z, U) - \eta(Z)\eta(U), \tag{7.9}$$

for any $U, Z \in \Gamma(TM)$. Thus, for $\delta_5 = \xi$, $M(\phi, \xi, \eta, g)$ defines an almost contact metric manifold. The 1-form η is closed.
We have

$$\Omega\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial z}\right) = g\left(\frac{\partial}{\partial x}, \phi\frac{\partial}{\partial z}\right) = g\left(\frac{\partial}{\partial x}, -\frac{\partial}{\partial x}\right) = -e^{2v}. \tag{7.10}$$

Hence we obtain $\Omega = -e^{2v}dx \wedge dz$. Thus, $d\Omega = -2e^{2v}dv \wedge dx \wedge dz = 2\eta \wedge \Omega$. Therefore, $M(\phi, \xi, \eta, g)$ is an almost Kenmotsu manifold. It can be seen that $M(\phi, \xi, \eta, g)$ is normal. So, it is a Kenmotsu manifold.
Then we have
$[\delta_1, \delta_2] = [\delta_1, \delta_3] = [\delta_1, \delta_4] = [\delta_2, \delta_3] = 0, [\delta_1, \delta_5] = \delta_1,\ [\delta_4, \delta_5] = \delta_4, [\delta_2, \delta_4] = [\delta_3, \delta_4] = 0, [\delta_2, \delta_5] = \delta_2, [\delta_3, \delta_5] = \delta_3$.
The Levi-Civita connection ∇ of the metric tensor g is given by Koszul's formula which is given by

$$2g(\nabla_X Y, Z) = Xg(Y,Z) + Yg(X,Z) - Zg(X,Y) - g(X,[Y,Z]) - g(Y,[X,Z]) + g(Z,[X,Y]). \tag{7.11}$$

Taking $\delta_5 = \xi$ and using Koszul's formula we get the following
$\nabla_{\delta_1}\delta_1 = -\delta_5, \nabla_{\delta_1}\delta_2 = 0, \nabla_{\delta_1}\delta_3 = 0, \nabla_{\delta_1}\delta_4 = 0, \nabla_{\delta_1}\delta_5 = \delta_1,$
$\nabla_{\delta_2}\delta_1 = 0, \nabla_{\delta_2}\delta_2 = -\delta_5, \nabla_{\delta_2}\delta_3 = 0, \nabla_{\delta_2}\delta_4 = 0, \nabla_{\delta_2}\delta_5 = \delta_2,$
$\nabla_{\delta_3}\delta_1 = 0, \nabla_{\delta_3}\delta_2 = 0, \nabla_{\delta_3}\delta_3 = -\delta_5, \nabla_{\delta_3}\delta_4 = 0, \nabla_{\delta_3}\delta_5 = \delta_3,$
$\nabla_{\delta_4}\delta_1 = 0, \nabla_{\delta_4}\delta_2 = 0, \nabla_{\delta_4}\delta_3 = 0, \nabla_{\delta_4}\delta_4 = -\delta_5, \nabla_{\delta_4}\delta_5 = \delta_4,$
$\nabla_{\delta_5}\delta_1 = \nabla_{\delta_5}\delta_2 = \nabla_{\delta_5}\delta_3 = \nabla_{\delta_5}\delta_4 = \nabla_{\delta_5}\delta_5 = 0.$

By the above results, we can easily obtain the non-vanishing components of the curvature tensor with respect to the Levi-Civita connection are as follows:
$R(\delta_1, \delta_2)\delta_2 = R(\delta_1, \delta_3)\delta_3 = R(\delta_1, \delta_4)\delta_4 = R(\delta_1, \delta_5)\delta_5 = -\delta_1,$
$R(\delta_1, \delta_2)\delta_1 = \delta_2, R(\delta_1, \delta_3)\delta_1 = R(\delta_5, \delta_3)\delta_5 = R(\delta_2, \delta_3)\delta_2 = \delta_3,$
$R(\delta_2, \delta_3)\delta_3 = R(\delta_2, \delta_4)\delta_4 = R(\delta_2, \delta_5)\delta_5 = -\delta_2, R(\delta_3, \delta_4)\delta_4 = -\delta_3,$
$R(\delta_2, \delta_5)\delta_2 = R(\delta_1, \delta_5)\delta_1 = R(\delta_4, \delta_5)\delta_4 = R(\delta_3, \delta_5)\delta_3 = \delta_5,$
$R(\delta_1, \delta_4)\delta_1 = R(\delta_2, \delta_4)\delta_2 = R(\delta_3, \delta_4)\delta_3 = R(\delta_5, \delta_4)\delta_5 = \delta_4.$

With the help of the above results we get the components of the Ricci tensor as follows:

$$S(\delta_1, \delta_1) = S(\delta_2, \delta_2) = S(\delta_3, \delta_3) = S(\delta_4, \delta_4) = S(\delta_5, \delta_5) = -4. \tag{7.12}$$

In this case from (3.3), we have $\alpha = 0$. Thus the manifold does not admit proper α-ARS. Also, the Ricci tensor is of Codazzi type and cyclic parallel. Hence the Theorem 4.1, Theorem 5.1 and Theorem 6.1 are verified.

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