POD analysis of large-scale structures through DNS of turbulent plane Couette flow

T. Tsukahara[1], K. Iwamoto[1,2] and H. Kawamura[1]

[1] Dept. of Mech. Eng., Tokyo Univ. of Science,
2641 Yamazaki, Noda, Chiba 278-8510, Japan tsuka@rs.noda.tus.ac.jp
[2] Present address: Dept. of Mech. Sys. Eng., Tokyo Univ. of A & T,
2-24-16 Nakacho, Koganei, Tokyo 184-8588, Japan iwamotok@cc.tuat.ac.jp

1 Introduction

Turbulent plane Couette flow has been extensively investigated, since this is one of the canonical flow cases. In addition, its monotonic velocity profile gives a significantly different character to the flow in the core region, as compared to a pressure-driven channel flow. In the Couette flow, it has been revealed that the central part contains large-scale low and high speed regions (large-scale structure, LSS) extending over a very long streamwise distance. Recently, authors' group [1] carried out the DNS at the moderate Reynolds numbers with applying the large computational domain. We reported that the finite-length LSS is captured when the streamwise domain size is larger than 64δ.

In this paper, the turbulent plane Couette flow is analyzed by means of the proper orthogonal decomposition (POD) in order to extract three-dimensional spatial modes from a flow field obtained from DNS. Moehlis *et al.* [2] and Holstad *et al.* [3] used POD to extract spatial modes from DNS of the Couette flow at low Reynolds numbers with relatively small domains. The present objective is to examine POD modes at low and moderate Reynolds numbers through the large-scale DNS, with emphasis on their Reynolds-number dependence.

2 Numerical condition

The flow is fully developed turbulent plane Couette flow as shown in Fig. 1. Calculations are performed at the Reynolds numbers $Re_w = 4U_w\delta/\nu = 3000$ and 8600. Details of the numerical scheme for the flow field can be found in the previous paper [1]. Instantaneous velocity field can be decomposed as a linear superposition of the eigenfunctions $\phi^{\mathbf{k}}$, where each eigenfunction is specified with a triplet $\mathbf{k} = (k_x, k_z, q)$. Note that an eigenvalue $\lambda^{\mathbf{k}}$ represents the energy in each POD mode of $\phi^{\mathbf{k}}$, so that the eigenfunctions can be sorted according to their contributions to the turbulent kinetic energy.

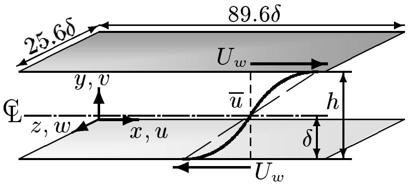
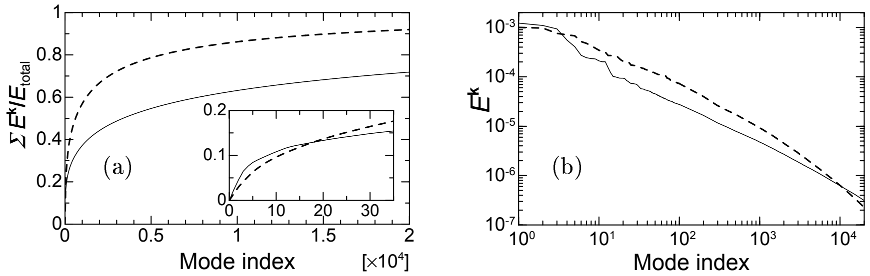


Fig. 1. Configuration of plane Couette flow. The periodic boundary condition is imposed in the horizontal directions.

Table 1. Eigenvalues of the POD modes.

	$Re_w = 3000$		$Re_w = 8600$	
Index	(k_x,k_z,q)	$E^{\mathbf{k}}_{\%}$	(k_x,k_z,q)	$E^{\mathbf{k}}_{\%}$
1	$(1,5,1)$	1.54	$(1,6,1)$	2.45
2	$(0,6,1)$	1.49	$(1,5,1)$	2.19
3	$(1,7,1)$	1.17	$(2,5,1)$	1.87
4	$(1,6,1)$	1.09	$(0,5,1)$	1.11
5	$(0,7,1)$	1.04	$(1,7,1)$	0.81



*footnote (5 July 2007) – – – –, $Re_w = 3000$; ———, $Re_w = 8600$.

Fig. 2. (a) Cumulative energy summation *versus* a number of POD modes. (b) Energy content $E^{\mathbf{k}}(=\lambda^{\mathbf{k}})$, non-dimensionalized by U_w and δ, of each single mode.

3 Result

A listing of the first fifth modes is presented in Table 1 with a comparison to the different Reynolds number simulation with the same box size. Here, $E^{\mathbf{k}}_{\%} = (E^{\mathbf{k}}/E_{\mathrm{total}})$ is the percentage of average total energy contained in the (k_x, k_z, q) mode. As shown in Table 1, the energetic POD modes are characterized by the low wavenumbers of $k_x = 0$–2 and $k_z = 5$–7. Especially, the first and second POD modes for $Re_w = 8600$ represent large energy fractions. Their spatial wavelengths of 90δ and 4.2–5.1δ in x and z directions are in good agreement with the scales of the finite-length LSS observed in the previous study with the two-point velocity correlation (refer Fig. 3 in [1]).

The cumulative energy sums for each Reynolds number are compared in Fig. 2 (a). The contributions of the first several modes are much higher than those of the following modes. We also observe that the convergence of the cumulative contribution of POD modes towards the 100% is relatively slow in the case of the higher Reynolds number. However, if emphasis is placed on the energy sum of the 1–15 dominant POD modes, the value for $Re_w = 8600$ is larger than that for the lower Re_w. It is due to the significant contribution of the first three modes for $Re_w = 8600$. In Fig. 2 (b), it is interesting to note that the energies of the first three POD modes are slightly increased (or unchanged) with the increasing Re_w. Therefore the first conclusion drawn from the present result is that the energy of the dominant POD modes (associated with the LSS) is of constant, at least in the range of the present Reynolds numbers.

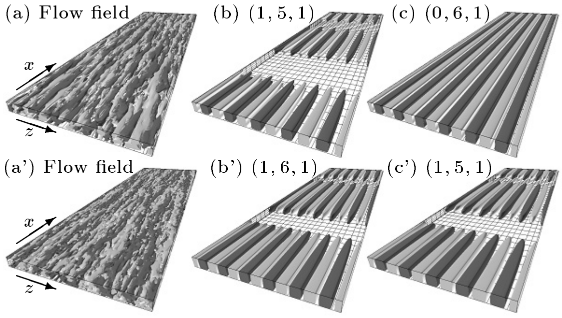


Fig. 3. Iso-surface of u' of instantaneous flow field (a, a'), and of u' from reconstruction of the first (b, b') and the second POD modes (c, c'). Light gray, positive; gray, negative. (a–c) $Re_w = 3000$, (a'–c') 8600.

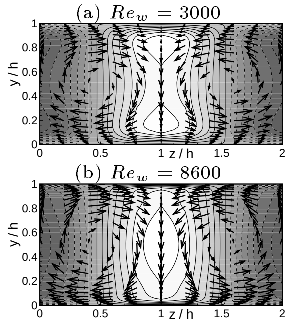


Fig. 4. Flow field of the energetic POD mode. Vector, v' and w'; contour, u' (negative contour is dashed).

For $Re_w = 8600$, the large-scale staggered pattern similar to the energetic modes is observed in an actual flow realization (see Figs. 3 (a'–c')). On the other hand, the second mode for $Re_w = 3000$ is characterized by $(0, 6, 1)$ without streamwise dependence. This elongated structure exceeds the domain, and its energy contribution is as much as the first mode. Here, if you look at the longitudinal two-point correlation coefficient $R_{uu}(\Delta x)$ (not shown here, please refer to [1]), it gradually decreases to zero at $Re_w = 3000$, while the $R_{uu}(45\delta)$ becomes negative at $Re_w = 8600$. This tendency is consistent with the experiment study [4]. This observation indicates the Reynolds-number dependence of the dominant structure. In the (z, y) plane, Fig. 4 shows the vectors of the most energetic mode and the counter-rotating streamwise vortices. The significant Reynolds-number dependence of the u' distribution is found: its magnitude for $Re_w = 3000$ is largest at $y^+ = 15$–20, while the peak for the higher Re_w is located at the channel center. In consequence, the present result implies that the dominant modes at $Re_w = 3000$ are essentially different from the modes of LSS at $Re_w = 8600$ due to the low-Reynolds-number effect.

This work was conducted in Research Center for the Holistic Computational Science (Holcs) supported by MEXT. The first author (T.T.) is granted by JSPS Fellowship (18-81). The computations were performed with the supercomputer at Information Synergy Center of Tohoku University.

References

1. T. Tsukahara, H. Kawamura, K. Shingai: *J. Turbulence* **7**, No. 19 (2006)
2. J. Moehlis, T.R. Smith, P. Holmes, H. Faisst: *Phys. Fluids* **14**, 2493 (2002)
3. A. Holstad, P.S. Johansson, H.I. Andersson, B. Pettersen: In *Proc. Sixth Int. ERCOFTAC Workshop on Direct and Large-Eddy Simulation* (edited by E. Lamballais, R. Friedrich, B.J. Geurts, O. Metais), Springer, pp 172–173 (2005)
4. N. Tillmark: PhD Thesis, Royal Institute of Technology, Stockholm (1995)