OSAKA UNIVERSITY

# Measuring Indirect Effects of Advertising Through Mass and Social Media

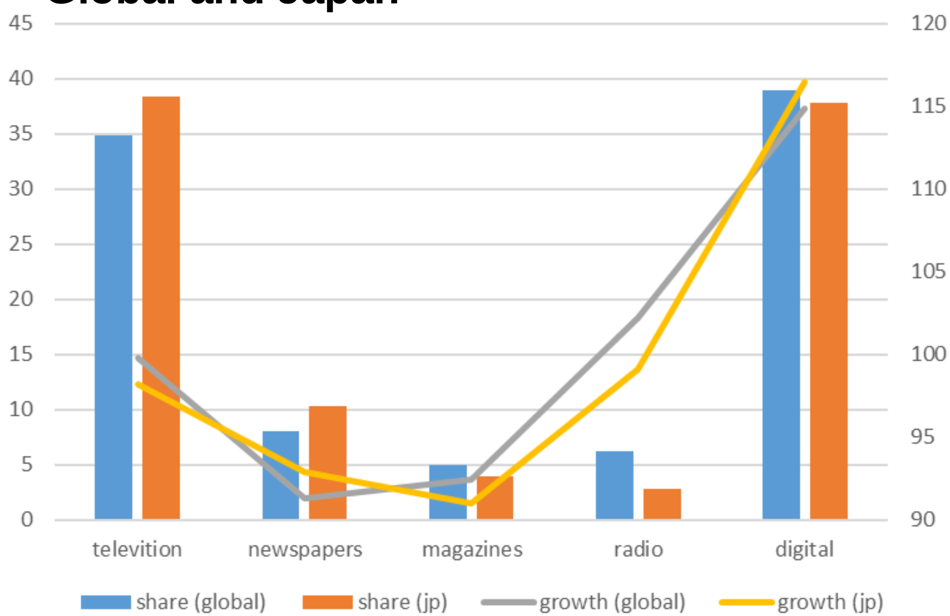
Dec. 6, 2019

Sotaro Katsumata (Osaka University)
Akihiro Nishimoto (Kwansei Gakuin University)
Eiji Motohashi (Yokohama National University)

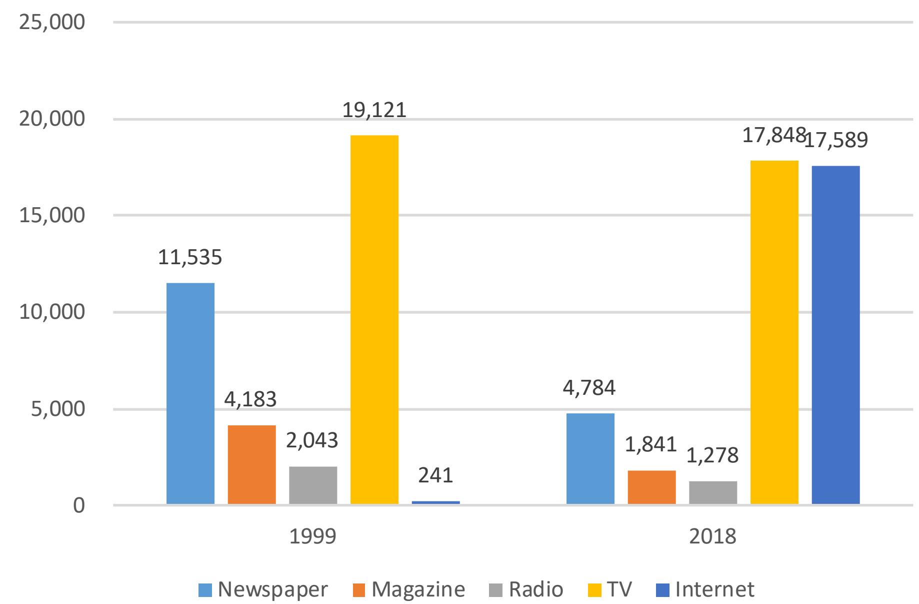
# Digital Shift in Advertising

- In 2018, global digital ad expenditure exceeded television ad spend
  - Media share: TV 34.9% (Growth -0.2%); digital 39.0% (growth 14.9%)
- Similar tendency is observed in Japan

**Ad expenditure and growth (2018) Global and Japan**




**Ad expenditure 1999 and 2018 (Japan)**




Sources: Dentsu (2019), Dentsu Aegis Network (2018, 2019)

# Digital Media

- Growth of Internet ad expenditure = growth of digital media
  - Growth of information traffic (OECD 2019)
- **Social** data
  - Lamberton and Stephen (2016), Kannan and Li (2017)
  - Social data reflects the performances of brands and firms
    - Wedel and Kannan (2017), Balducci and Marinova (2018)
  - Taxonomy of digital media (Stephen and Galak 2012)
    - **Paid media**: display/banner ads, search ads,
    - **Owned media**: company and brand websites
    - **Earned media**: SNS posts, online reviews
  - Growth of **earned media**

# How to Use TV Ads?

- Fall in the number of young TV viewers
  - *Marketing Charts* (2019)
  - https://www.marketingcharts.com/featured-105414
- Split among online/offline media
  - Du, Joo, and Wilbur (2019)
- TV ads enhance communications
  - Vakratsas and Ma (2005), Naik and Raman (2003), Onishi and Manchanda (2012)

# Research Questions

- **Measuring the effect of mass-media (TV) adverting including indirect effect via social media**
  - Measuring the ROI of TV ads in the social media era

- **Examine the different impacts of mass and social media**
  - Examine the difference in the carryover effect
  - Heterogeneous impact across segments

# From Mass to Social Media

- Mass media communications stimulates online behaviors and evaluations
- Synergy effect
  - Online and offline media
    - Vakratsas and Ma (2005), Naik and Raman (2003)
  - TV and social media
    - Fossen and Schweidel (2019)
- Offline ads affect online behavior
  - Inspire proactive online actions
    - Joo, Wilbur, Cowgill, and Zhu (2013), Joo, Wilbur, and Zhu (2016), Chandrasekaran, Srinivasan, and Sihi (2018)
  - Immediate effect
    - Du, Xu, and Wilbur (2019)

# Social Media and Market Performance

- Relation of number of online chatters related to market share
  - Independent of the the media (paid, owned, and earned)
  - Earned
    - -> Online sales: Chevalier and Mayzlin (2006)
    - -> Offline sales: Dellarocas, Zhang and Awad (2007), Onishi and Manchanda (2012), Liu (2006)
    - -> Sales: Duan et al.(2008)
    - Negative reviews -> stock price: Luo (2009)
  - Owned
    - F(Firm) Generated Contents -> Sales: Kumar et al. (2016)
  - Paid
    - Search ads -> sales: Fang, Huang, and Palmatier (2015)
- Online earned media
  - Low-cost or free
    - Berger (2014), Berger and Milkman (2012), Tirunillai and Tellis (2014)
  - Firms cannot control “quality” and “valence” of reviews
    - Floyed et al. (2014), You et al. (2015), Chevalier and Mayzlin (2006)

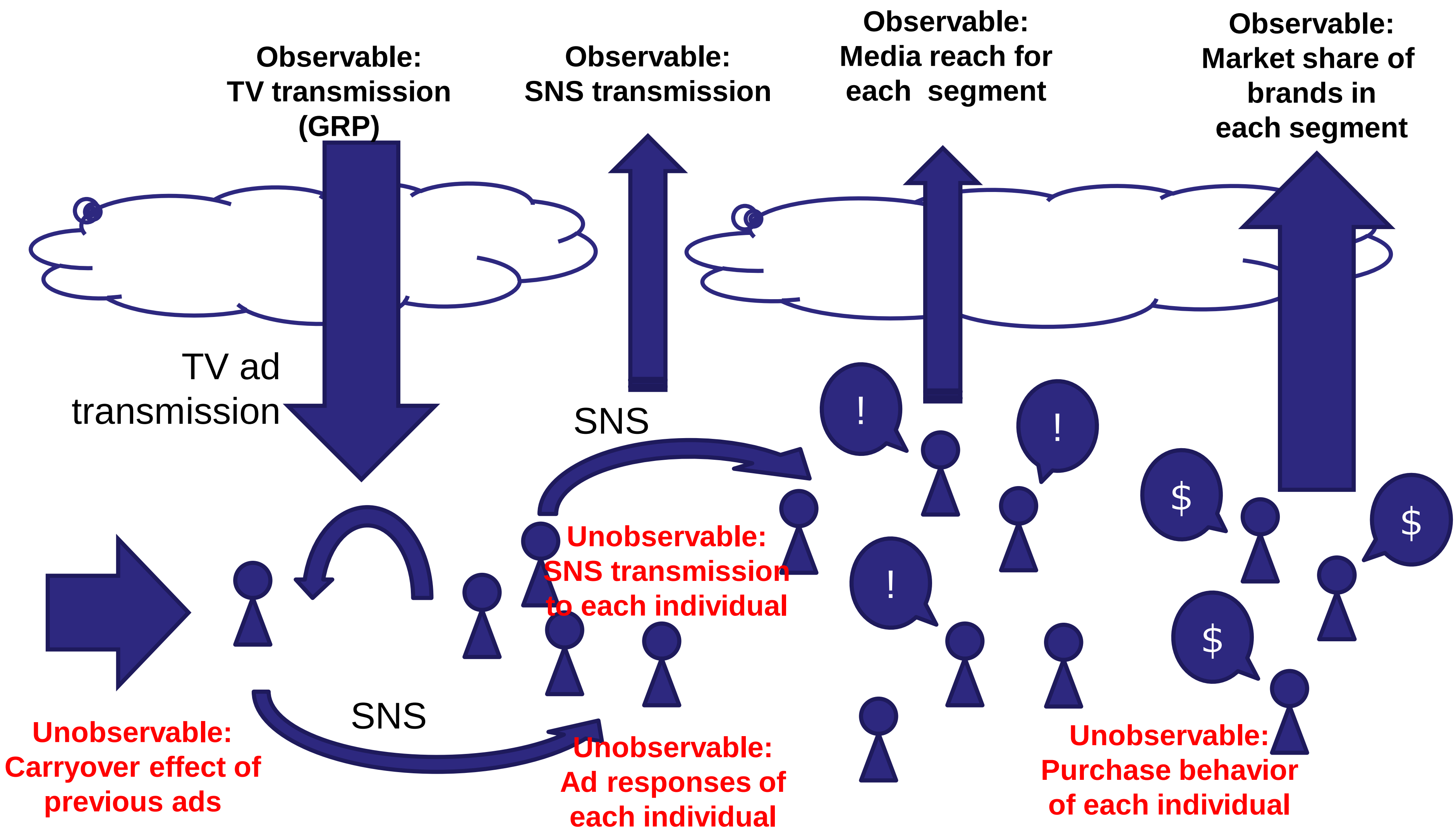
# Long-Term Effect

- In general, advertising effects are long-term
  - Assael (2011), Lodish et al. (1995), Tellis and Franses (2006)
- Examine by vector auto-regression (VAR) model
  - Offline: Joshi and Hanssens (2010)
  - Online: Borah and Tellis (2016)
  - Off and on: Du, Joo, and Wilbur (2019)
- Examine by decay parameter
  - Dube, Hitsch, and Manchanda (2005), Lopez and Zhu (2015)

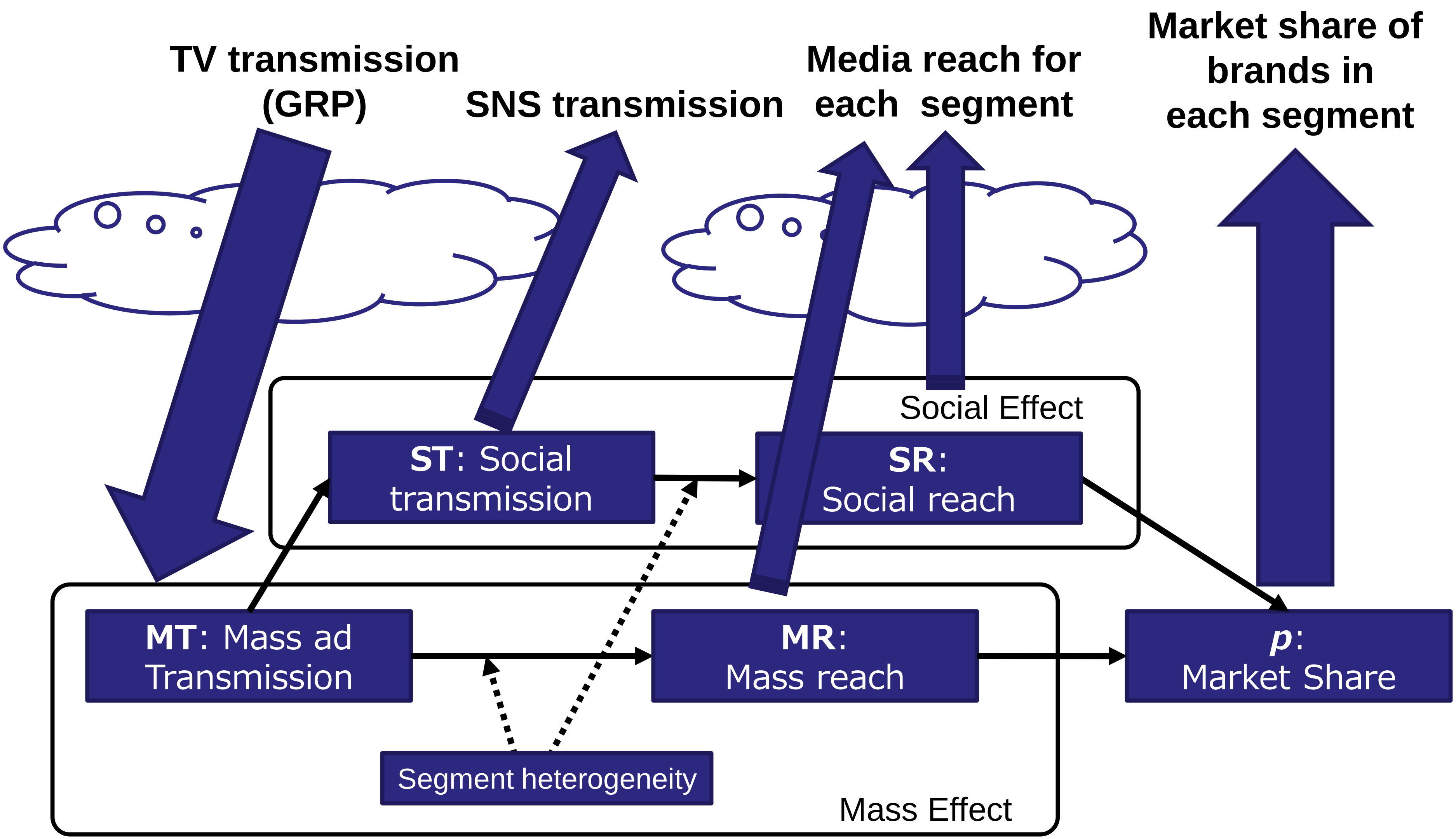
# Research Positioning

| | Mass -> Social (online) | Mass -> Performance | Social (online) -> Performance | Indirect Effect | Long-Term |
|---|---|---|---|---|---|
| **Dellarocas, Zhang and Awad (2007)** | | | Online review -> offline sales | | |
| **Chevalier and Mayzlin (2006)** | | | online WOM -> online sales | | |
| **Joo, Wilbur, Cowgill, and Zhu (2013)** | Offline Advertising -> Online Search | | | | |
| **Joo, Wilbur, and Zhu (2016)** | Offline Advertising -> Online Search | | | | |
| **Chandrasekaran, Srinivasan, and Sihi (2018)** | Offline Advertising -> Online Search | | | | |
| **Kummer et al. (2016)** | | Offline Advertising -> Sales | Online Owned Media -> Sales | On * Off -> Sales | |
| **Stephen and Galak (2012)** | | Traditional media -> Sales | Online Media -> Sales | | lagged effects |
| **Onishi and Manchanda (2012)** | Offline Advertising -> Online blog | Offline Advertising -> Offline Sales | Online blog -> Offline sales | Off -> On -> Sales | |
| **Du, Joo, and Wilbur (2019)** | | Offline Advertising -> Offline Sales | Online Advertising -> Offline Sales | | VAR |
| **Our Study** | Offline Advertising -> Online WOM | Offline Advertising -> Market Share | Online WOM -> Market Share | Off -> On -> MS & Off -> MS | Decay Parameter |

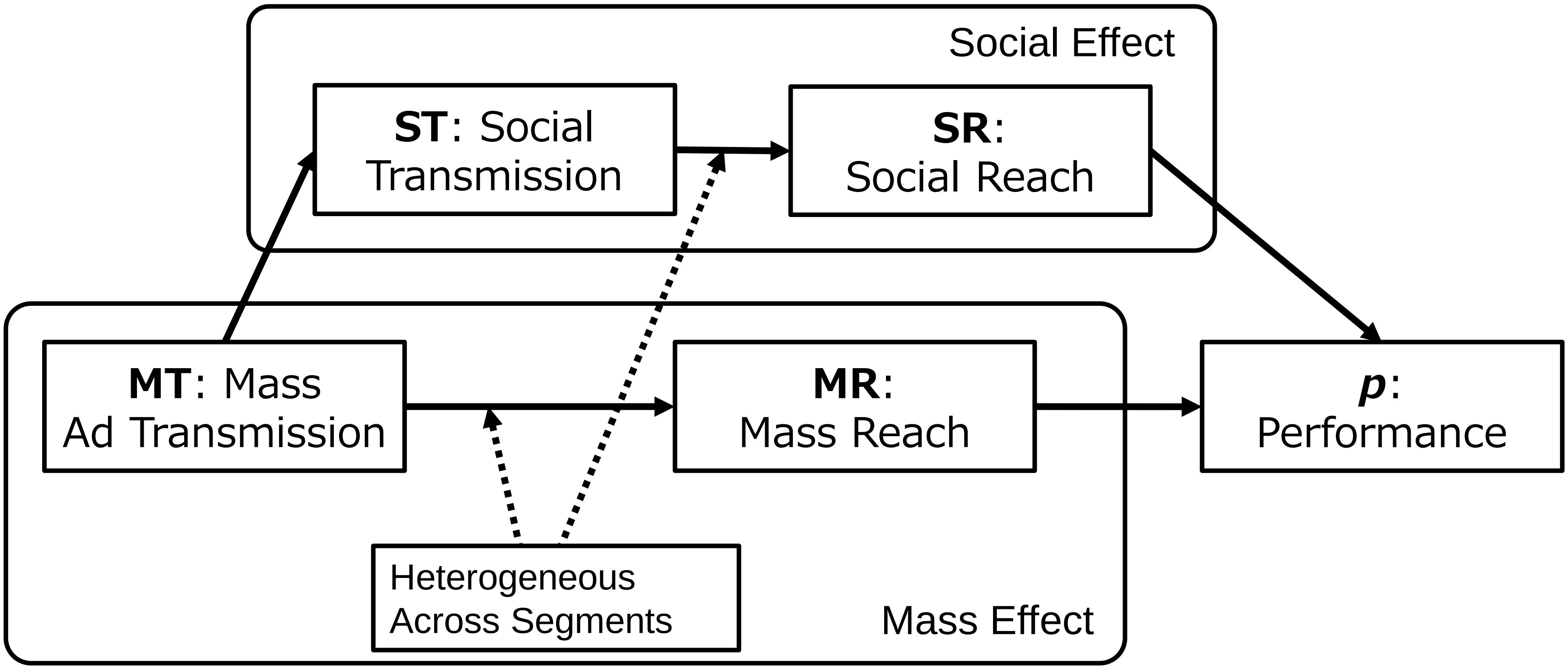
# From Mass Transmission to Market Performance



# From Mass Transmission to Market Performance



# Conceptual Model



# Demand Function

- Set up a model based on the aggregate demand function (BLP model)
  - Berry, Levinsohn, and Pakes (1995), Berry (1994)
  - Application: Nevo (2000), Duan and Mela (2009)
  - Bayesian Estimation: Jiang, Manchanda, and Rossi (2009)
- Utility function: utility of consumer $i$ for brand $j$ $(j = 0, \cdots, J)$ at time $t$

$$U_{ijt} = \mu_{ijt} + e_{ijt}$$

- Let the utility of outside goods (brand $0$) be $\mu_{i0t} = 0$

$$U_{i0t} = 0 + e_{i0t}$$

- Assume that $e_{ijt}$ follows the *Type I Extreme Value Distribution*, we obtain the following choice probability (e.g., Train 2009)

$$s_{ijt} = \frac{\exp(\mu_{ijt})}{1 + \sum_{k=1}^{J} \exp(\mu_{ijt})}$$

# Choice Probability

- Assuming that the components of utility $\mu_{ijt}$ as follows (Random coefficient model):

$$\mu_{ijt} = \alpha_0 + X_{sjt}\theta_i + \eta_{sjt}$$

  - $X_{sjt}$: marketing variables for segment $s$ to which consumer $i$ belongs
- Let $\theta_i = \beta + v_i, v_i \sim N(0, \Sigma)$, and $\eta_{sjt} \sim N(0, \tau^2)$; choice probability is defined as follows:

$$p_{ijt} = \frac{\exp\left(\alpha_0 + X_{sjt}\theta_i + \eta_{sjt}\right)}{1 + \sum_{k=1}^{J} \exp\left(\alpha_0 + X_{sjt}\theta_i + \eta_{sjt}\right)}$$

- Using the values of $\beta$ and $v_i$, we get:

$$p_{ijt} = \frac{\exp\left(\alpha_0 + X_{sjt}(\beta + v_i) + \eta_{sjt}\right)}{1 + \sum_{k=1}^{J} \exp\left(\alpha_0 + X_{sjt}(\beta + v_i) + \eta_{sjt}\right)}$$

  - Note that $E(v_i) = 0$, therefore, $E\left(X_{sjt} v_i\right) = 0$.

# Aggregate Demand

- Components of $\mu_{stj}$:

$$\mu_{sjt} = \alpha_0 + X_{sjt}\beta + \eta_{sjt}$$

- Choice probability:

$$p_{ijt} = \frac{\exp\left(\mu_{sjt} + X_{sjt}v_i\right)}{1 + \sum_{k=1}^{J} \exp(\mu_{skt} + X_{skt}v_i)}$$

- Since individual variance depends on $v_i$, we can obtain the aggregate demand (market share) introducing $\pi(v_i|\Sigma)$
  - Jiang, Manchanda, and Rossi (2009)
- Market share of segment $s$ is given by

$$p_{stj} = \int \left[\frac{\exp\left(\mu_{sjt} + X_{sjt}v_i\right)}{1 + \sum_{k=1}^{J} \exp(\mu_{skt} + X_{skt}v_i)}\right] \pi(v_i|\Sigma)dv_i, if\ i \in N_s$$

  - Note that $N_s$ is a set of consumers of segment $s$

# Model

- Model Y: market share of segment $s$

$$\mu_{sjt} = \alpha_0 + CMR_{sjt}\beta_{MR} + CSR_{sjt}\beta_{SR} + Price_{jt}\beta_{Pr} + \eta_{sjt}$$

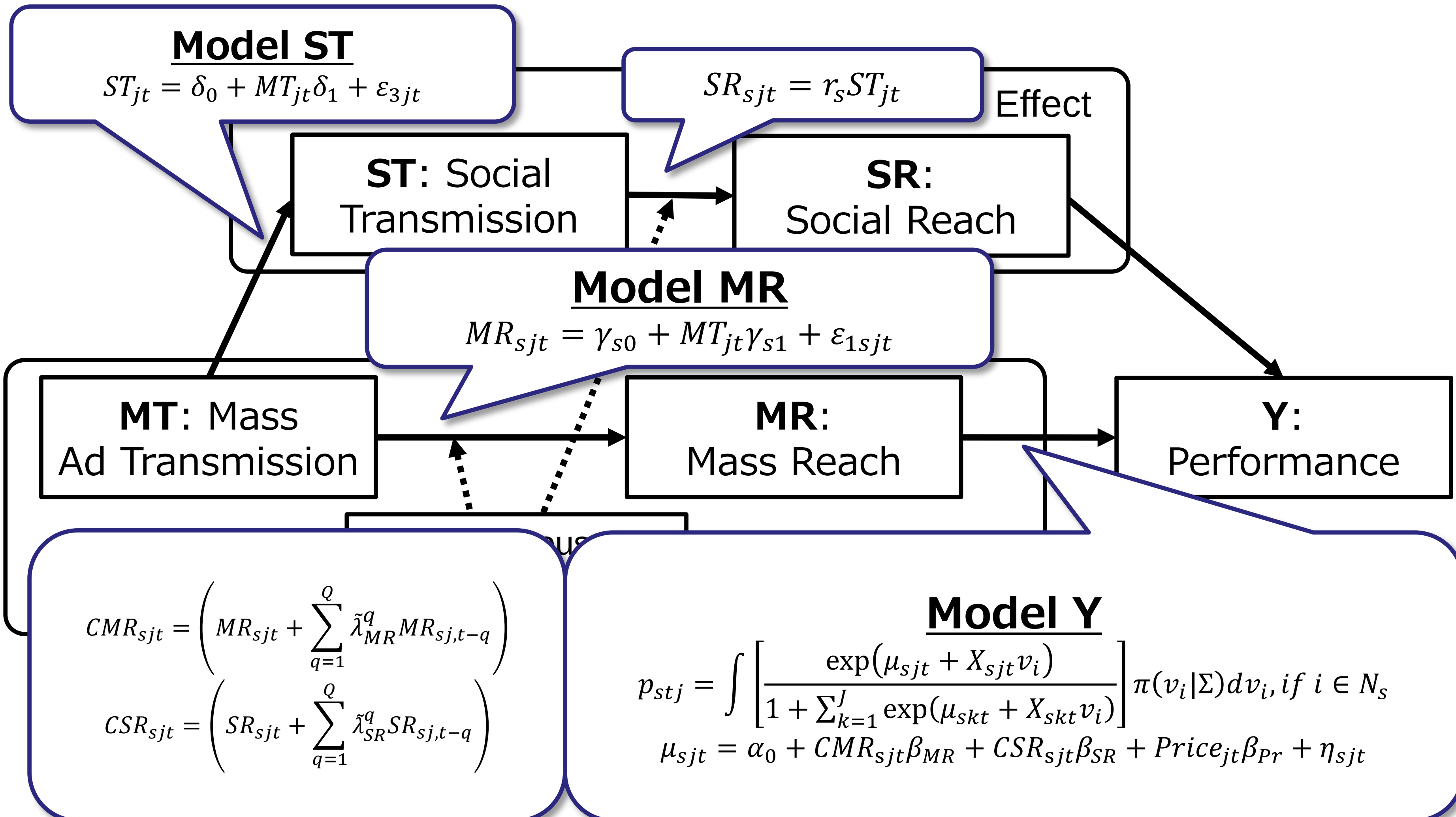
  – Carryover effect of advertising (Dube et al. 2005)

$$CMR_{sjt} = \left( MR_{sjt} + \sum_{q=1}^{Q} \tilde{\lambda}_{MR}^{q} MR_{sj,t-q} \right), \tilde{\lambda}_{MR} = \frac{\exp(\lambda_{MR})}{1 + \exp(\lambda_{MR})}$$

$$CSR_{sjt} = \left( SR_{sjt} + \sum_{q=1}^{Q} \tilde{\lambda}_{SR}^{q} SR_{sj,t-q} \right), \tilde{\lambda}_{SR} = \frac{\exp(\lambda_{SR})}{1 + \exp(\lambda_{SR})}$$

- Structural Equation Modeling
  – Model MR: $MR_{sjt} = \gamma_{s0} + MT_{jt}\gamma_{s1} + \varepsilon_{1sjt}$
  – Model ST: $ST_{jt} = \delta_0 + MT_{jt}\delta_1 + \varepsilon_{2jt}$
- Complementation from public data
  – Note that $r_s$ is the ratio of social media usage observed from public data
  – $SR_{sjt} = r_s ST_{jt}$

# Conceptual Model



# Indirect Effect

- Obtain MCMC simulation samples and examine the effects ($l$ indicates the $l$-th MCMC sample)
  - Cf. Yuan and McKinnon (2009)

| Effect | Expression |
|---|---|
| Indirect effect via mass media | $Mass^{(l)} = \beta_{MR}^{(l)}\gamma_{s1}^{(l)}$ |
| Indirect effect via social media | $Social^{(l)} = \beta_{SR}^{(l)} r_s \delta_1^{(l)}$ |
| Total Effect | $Total^{(l)} = Mass^{(l)} + Social^{(l)}$ |
| Carryover effect after q-weeks | $Mass_q^{(l)} = \left(\tilde{\lambda}_{MR}^{(l)}\right)^q Mass^{(l)}$<br>$Social_q^{(l)} = \left(\tilde{\lambda}_{SR}^{(l)}\right)^q Social^{(l)}$ |
| Infinite sum of carryover effect | $Mass_{Inf}^{(l)} = \frac{1}{1-\tilde{\lambda}_{MR}^{(l)}} Mass^{(l)}$<br>$Social_{Inf}^{(l)} = \frac{1}{1-\tilde{\lambda}_{SR}^{(l)}} Social^{(l)}$ |

# Empirical Analysis

- Product Category: Beer (Beer, low malt beer, new genre beer)
- Number of brands: J = 21
- Period of Analysis: T = 123 weeks + 26 weeks of carryover effect
  - Follows six month lag of Lopez et al. (2015)
  - from August 17, 2015 to December 18, 2017
    - Use data from February 16, 2015 to obtain the carryover effect
- Number of segments: S = 10
  - {Male, Female} x {20s, 30s, 40s, 50s, over 60s}
- Incorporating random effect intercepts
  - Model ST: brands
  - Model MR: brands and segments
  - Model Y: brands and segments
- Comparison Models
  - For Model Y, compare the model fitness with only MR, only SR, and without carryover effect models

# Data Description

| | Description |
|---|---|
| Y: Volume share | From *SCI database* collected and provided by *INTAGE, Inc*. Weekly sales volumes of each brand are recorded in the database. The volume share is divided by the sum of sales volume, and further divided by 1 minus the share of outside goods. |
| Share of outside goods | Obtain from CPI (Consumer Price Index) weight reported by *Statistics Bureau of Japan*. The sum of market share of objective brands is roughly 0.8. The expenditure weight of "beer," "low malt," and "new genre" categories are 53, 20, and 5, respectively, and the weight of all types of alcohol is 136. Therefore, the outside brand share is obtained from 1 − 0.8*(53+20+5)/136 = 0.541. This study assumes that the share of outside goods is invariant. |
| MT: Mass media transmission | The household TV program ratings (Kanto region) are Provided by *Video Research Ltd*. Let p be the rating; we define MT = log(p + 1) following Dube et al. (2005), and Lopez and Zhou (2015). |
| MR: Mass media reach | The TV program ratings for each segment (Kanto region) are provided by *Video Research Ltd*. Same as MT. Let $p_s$ be the segment rating, MR =log($p_s$ + 1) |
| ST: Social media transmission | Weekly number of posts of twitter, blogs, and online news. Let the number of posts be n; we obtain ST = log(n + 1). |
| SR: Social media reach | The usage rate of online media for each segment collected and reported by the communication usage trend survey of Ministry of Internal Affairs and Communications (MIC). Let the usage rate of segment s is $r_s$; SR = $r_s$*ST. |
| Price | Since consumer price index (CPI) reports the price level of "beer," "low malt," and "new genre," substitute these category prices for each corresponding brand. |

# Settings

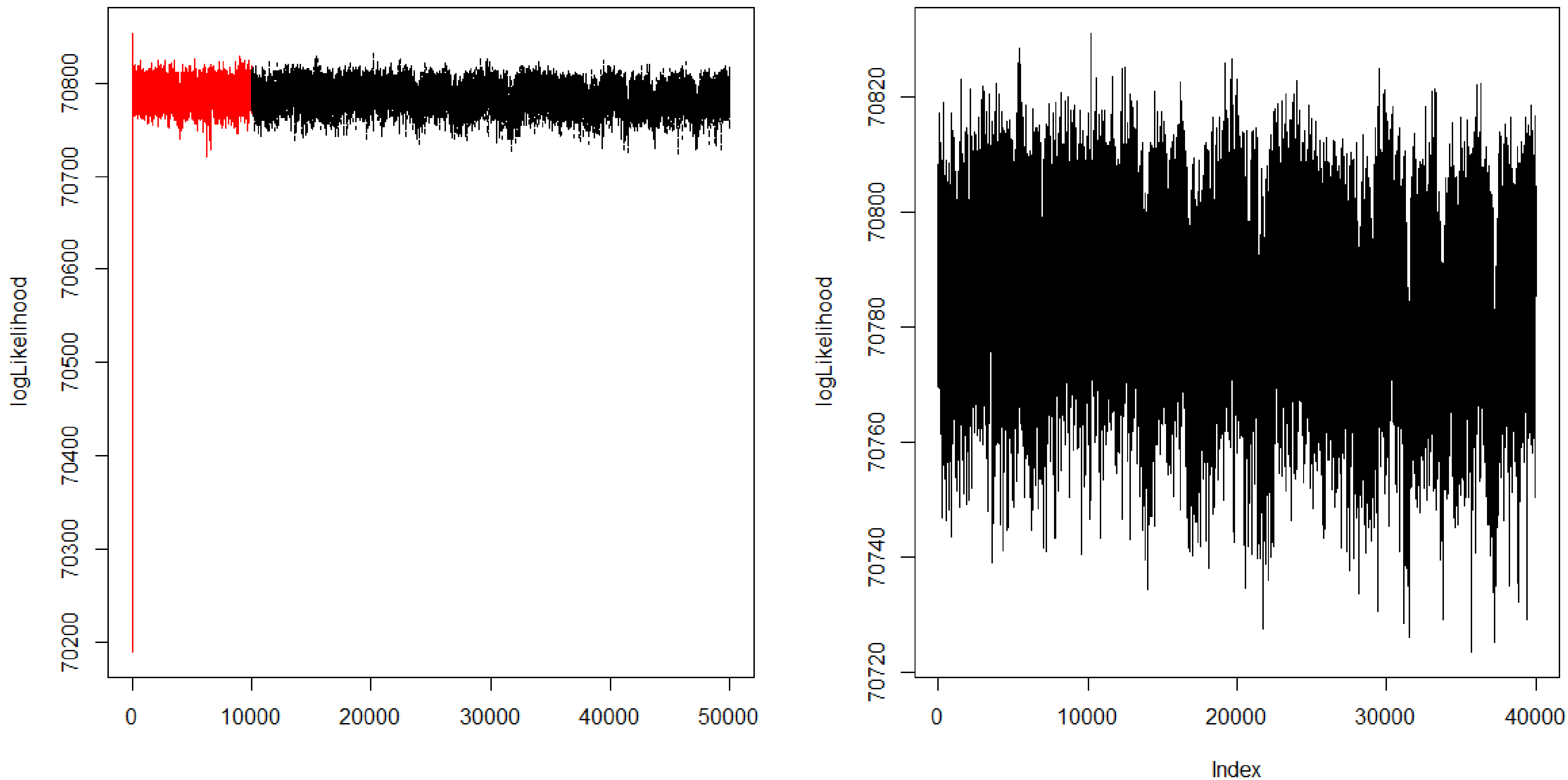
- Model Y
  - Distributions of parameters: $\{\alpha, \beta, \lambda, \tau^2, \Sigma\}$
    - $\alpha, \beta$: Multivariate normal
    - $\tau^2$: Gamma
    - $\lambda$: Random walk Metropolis-Hastings (standard deviation = 0.12).
    - $\Sigma$: Obtain the lower triangle matrix $R$ of Cholesky root ($RR' = \Sigma$) and estimate the elements of the matrix from random walk M-H (standard deviation = 0.004).
  - Number of virtual consumers of BLP
    - H = 50, following Jiang et al. (2009)
  - Initial Values
    - $\alpha, \beta, \lambda$: Obtain by averaging the OLS model provided by Berry (1994). The initial value of $\lambda$ is obtained by grid search, with interval of 0.1, from -4 to +4.
    - $\Sigma$: Let the diagonal elements of Cholesky root $\Sigma$ are $r_{ii} = \exp(-5)$.
    - $\tau^2$: 1
- Model ST, Model SR
  - MCMC Linear Model
- Number of iterations
  - Burn-in 10,000, Sampling 40,000

# Result of Model Comparison (Model Y)

Model Y

| | Mass Effect | Social Effect | Long-term Effect | Log Marginal Likelihood | logBF |
|---|---|---|---|---|---|
| Comparison Model 1 | ✓ | | | 70415.110 | 370.88 |
| Comparison Model 2 | | ✓ | | 70530.200 | 255.79 |
| Comparison Model 3 | ✓ | ✓ | | 70545.280 | 240.71 |
| Comparison Model 4 | ✓ | | ✓ | 70554.500 | 231.49 |
| Comparison Model 5 | | ✓ | ✓ | 70729.830 | 56.16 |
| Proposed Model | ✓ | ✓ | ✓ | 70785.990 | - |

# Sampling Path(Model Y)



# Result: Model Y

| Model Y | Post.Mean | Post.Median | Post.sd | 2.5% | 97.5% | HPD |
|---|---|---|---|---|---|---|
| Marketing Variables | | | | | | |
| Mass Effect ($\beta_{MR}$) | 0.100 | 0.100 | 0.028 | 0.056 | 0.158 | *** |
| Social Effect ($\beta_{SR}$) | 0.187 | 0.186 | 0.031 | 0.130 | 0.250 | *** |
| Price ($\beta_{Price}$) | -4.503 | -4.504 | 0.449 | -5.373 | -3.622 | *** |
| Decay Parameters | | | | | | |
| Mass Decay ($\lambda_{MR}$) | 0.959 | 0.964 | 0.035 | 0.882 | 0.999 | *** |
| Social Decay ($\lambda_{SR}$) | 0.784 | 0.790 | 0.051 | 0.671 | 0.872 | *** |

*Note) +:10%, *: 5%, **: 1%, ***: 0.1% HPDI does not include 0. Acceptance rate of λ is 0.201.*

| Σ | Mass Effect | Social Effect | Price |
|---|---|---|---|
| Mass Effect | 0.00003 | -0.00001 | -0.00005 |
| Social Effect | -0.00001 | 0.00002 | -0.00023 |
| Price | -0.00005 | -0.00023 | 0.00436 |

*Note) Italic: 10%, bold: 5% HPDI does not include 0. Acceptance rate of Σ is 0.268.*

# Result: Model ST, Model MR

| Model ST | Post.Mean | Post.Median | Post.sd | 2.5% | 97.5% | HPD |
|---|---|---|---|---|---|---|
| δ (MT: Mass transmission) | 0.342 | 0.342 | 0.028 | 0.286 | 0.398 | *** |

| Model MR | Post.Mean | Post.Median | Post.sd | 2.5% | 97.5% | HPD |
|---|---|---|---|---|---|---|
| γ (MT: Mass transmission) | | | | | | |
| M20 | 0.494 | 0.494 | 0.003 | 0.489 | 0.499 | *** |
| M30 | 0.662 | 0.662 | 0.002 | 0.657 | 0.666 | *** |
| M40 | 0.739 | 0.739 | 0.002 | 0.736 | 0.743 | *** |
| M50 | 0.714 | 0.714 | 0.002 | 0.710 | 0.717 | *** |
| M60 | 0.799 | 0.799 | 0.002 | 0.796 | 0.803 | *** |
| F20 | 0.681 | 0.681 | 0.003 | 0.676 | 0.686 | *** |
| F30 | 0.729 | 0.729 | 0.002 | 0.725 | 0.733 | *** |
| F40 | 0.803 | 0.803 | 0.002 | 0.800 | 0.807 | *** |
| F50 | 0.881 | 0.881 | 0.001 | 0.878 | 0.883 | *** |
| F60 | 0.891 | 0.891 | 0.002 | 0.888 | 0.894 | *** |

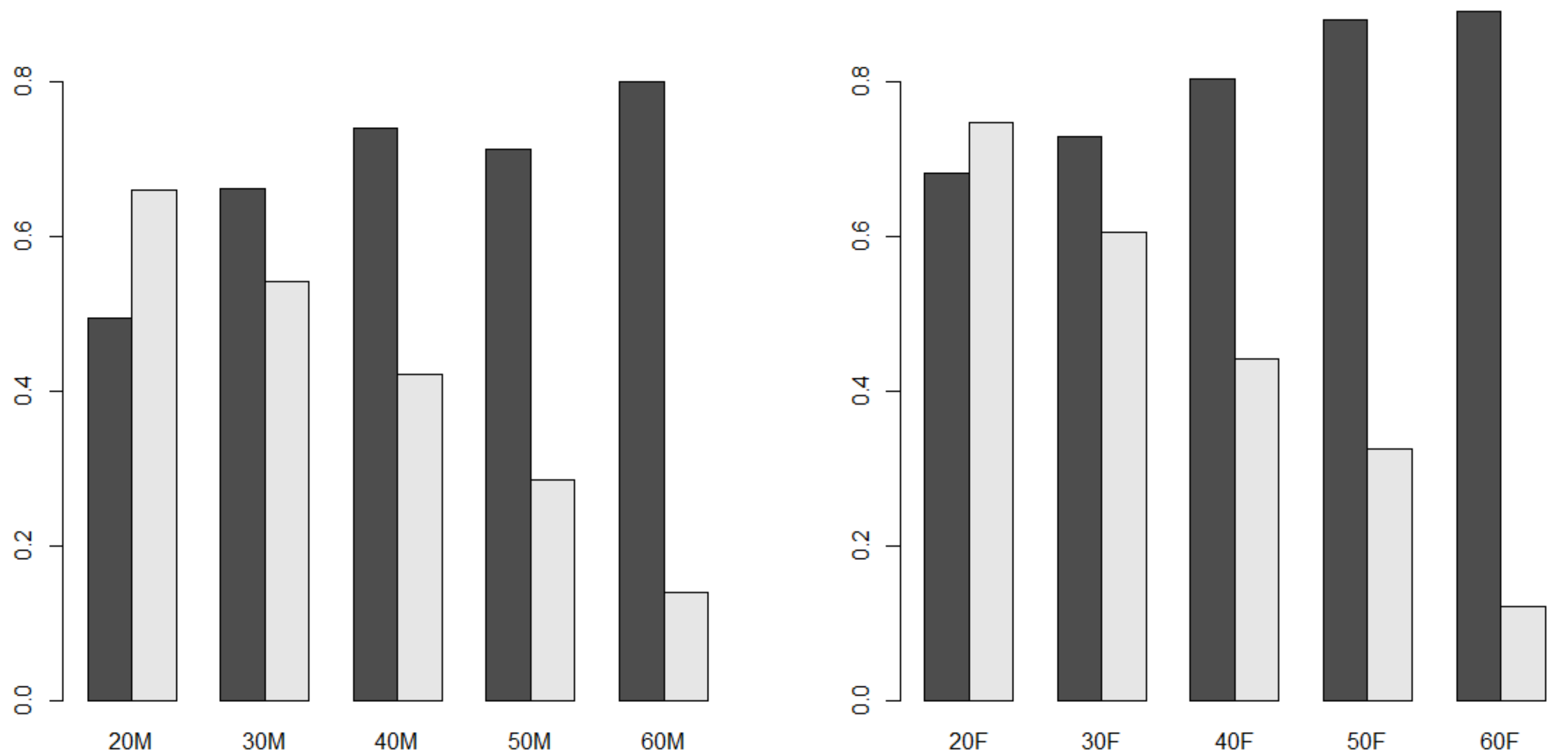
*Note) +:10%, *: 5%, **: 1%, ***: 0.1% HPDI does not include 0.*

# Overview of the Result

- Model Y
  - CMR (cumulative mass effect)：positive
  - CSR (cumulative social effect)：positive
  - Price: negative
  - Mass decay: weekly 0.959 (4 weeks: 0.846)
  - Social decay: weekly 0.784 (4 weeks: 0.378)
    - Mass carryover effect is larger than social
- Model ST
  - MT (mass transmission): positive
- Model SR
  - In general, male < female, younger < older

# Transmission Vs. Reach

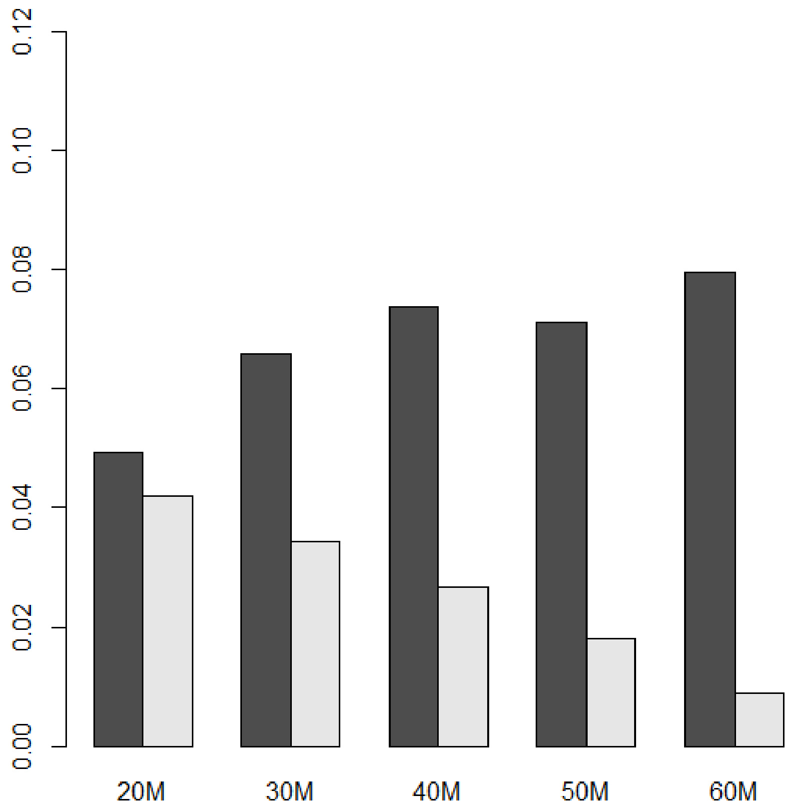
- ■ Mass Reach (*γ*) vs. □ Social Reach (*r*)

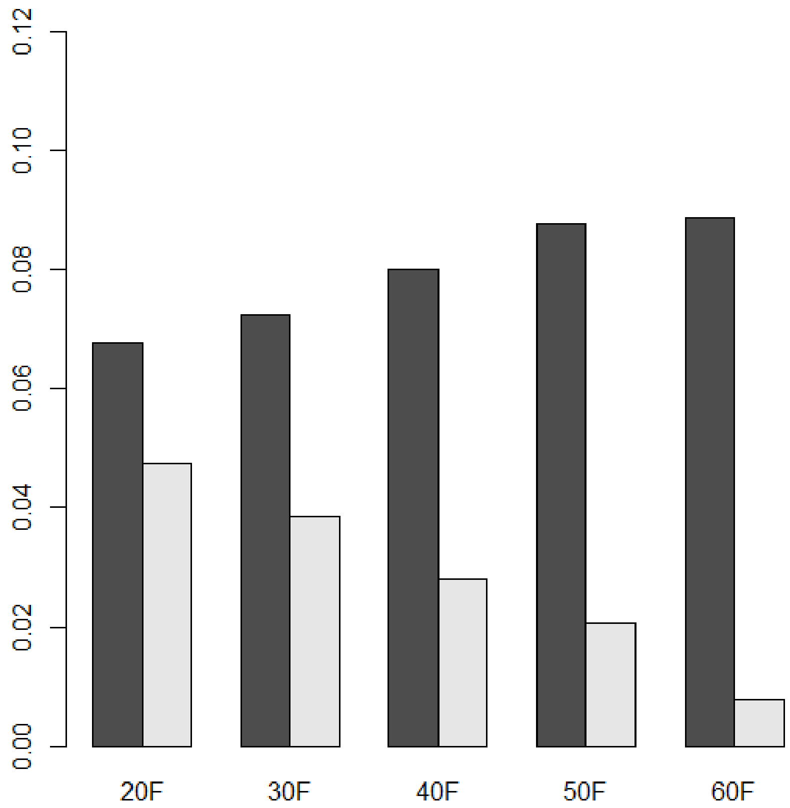


# Indirect Effect of the First Week

■: Indirect effect via mass media

■: Indirect effect via social media

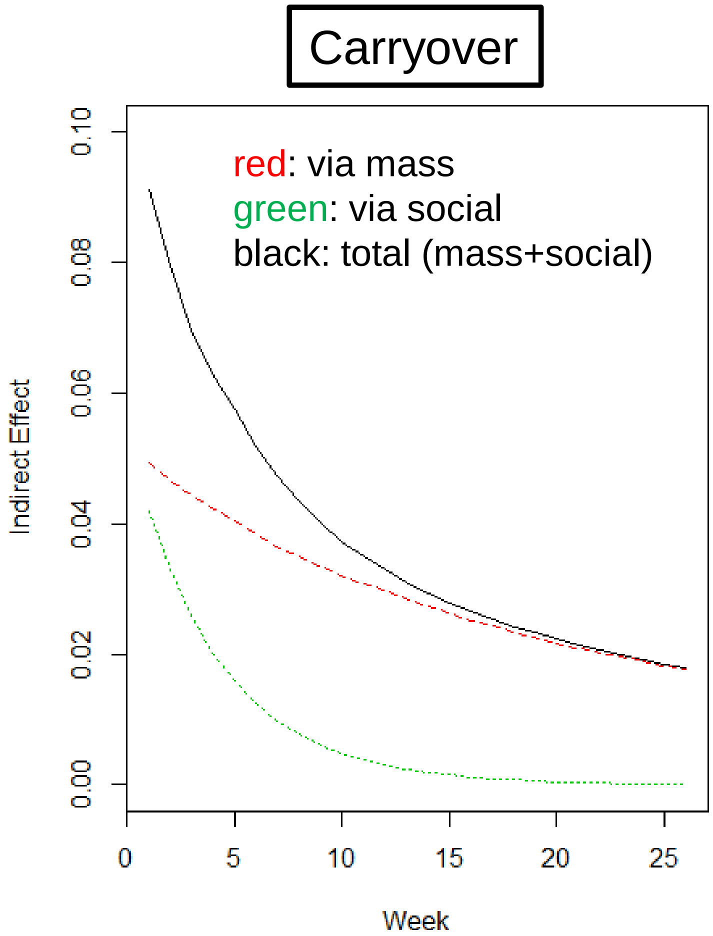


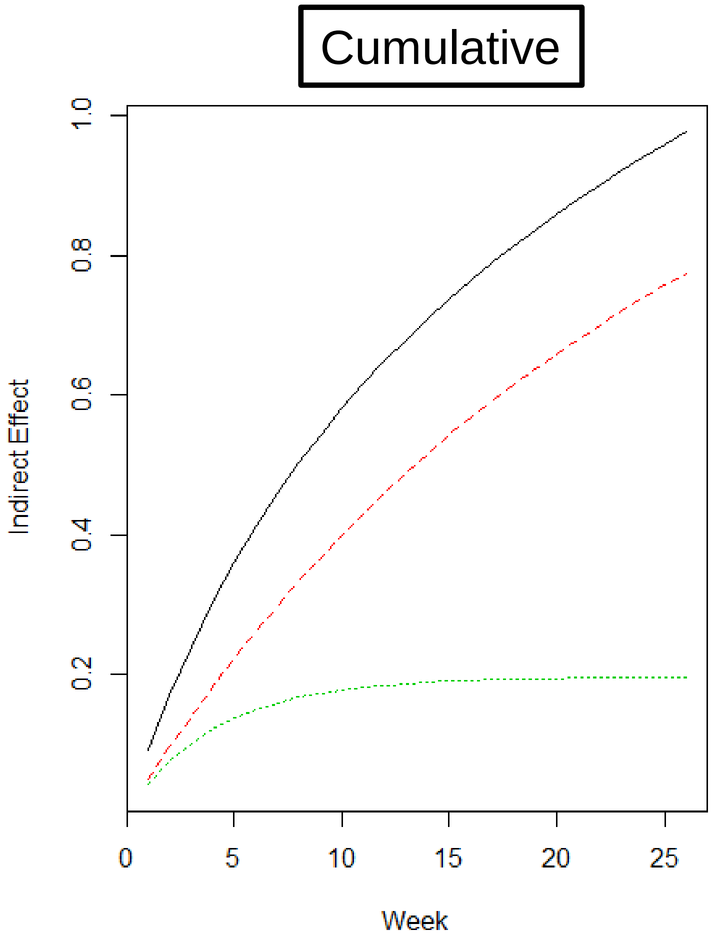



# Indirect Effect

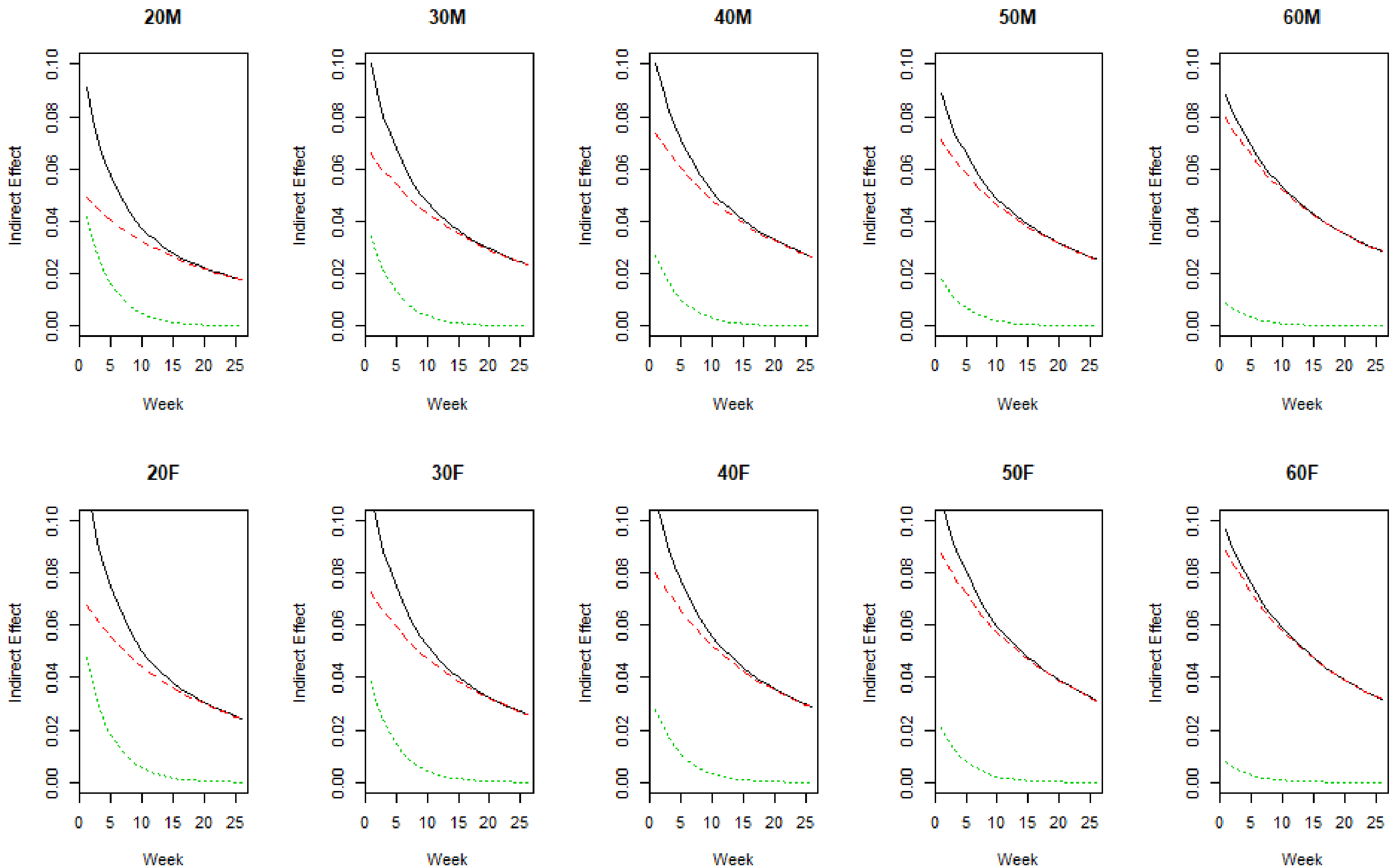
- Indirect effect via mass media
  - Effect of MT (Mass transmission) -> MR (Mass reach) -> Y (Market share)
- Indirect effect via social media
  - Effect of MT (Mass transmission) -> ST (Social transmission) -> SR (Social reach) -> Y (Market share)
- Total indirect effect
  - Sum of the indirect effect via mass and social media

# Carryover/Cumulative Effect (20s, Male)

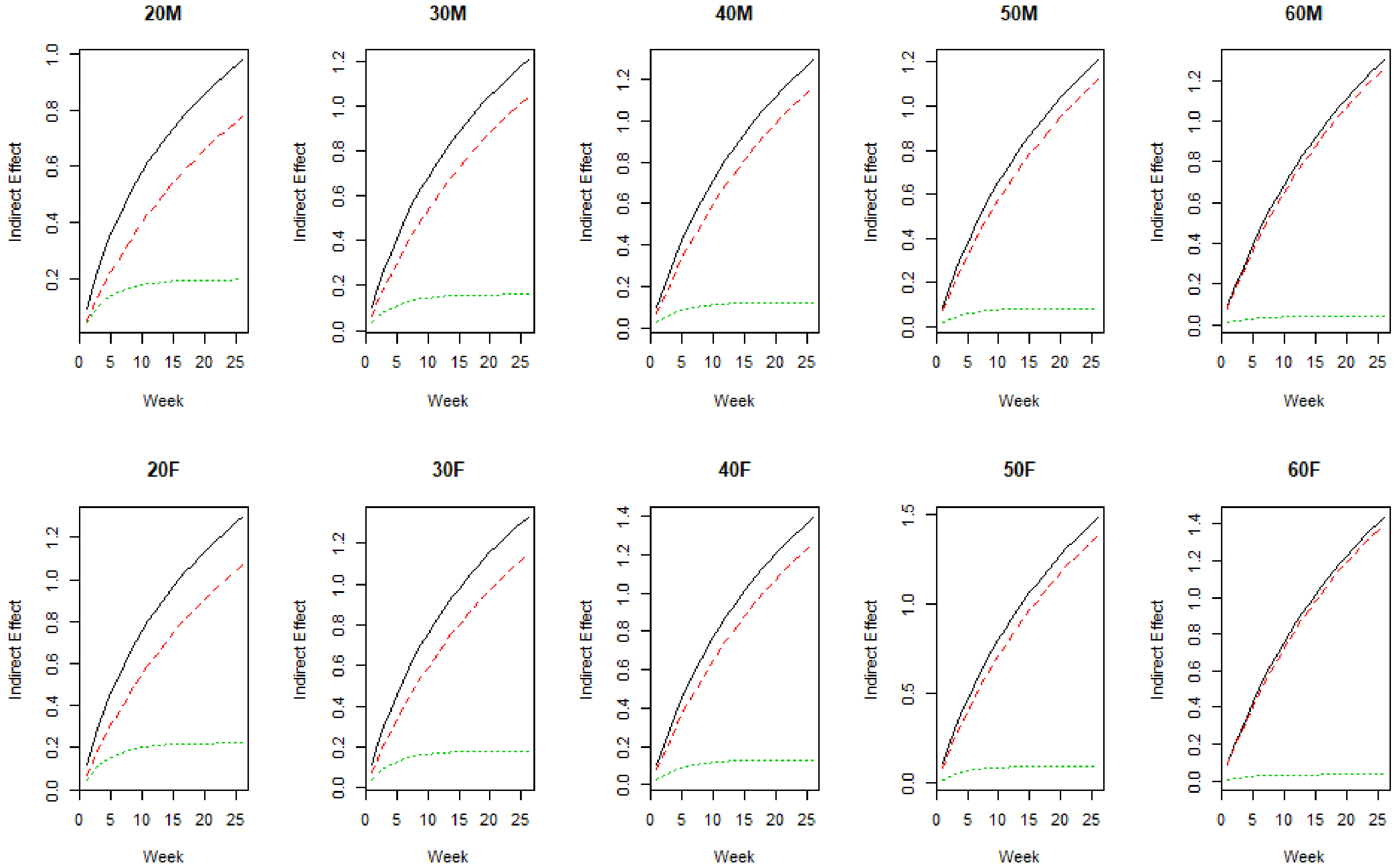


# Segment Carryover Effect



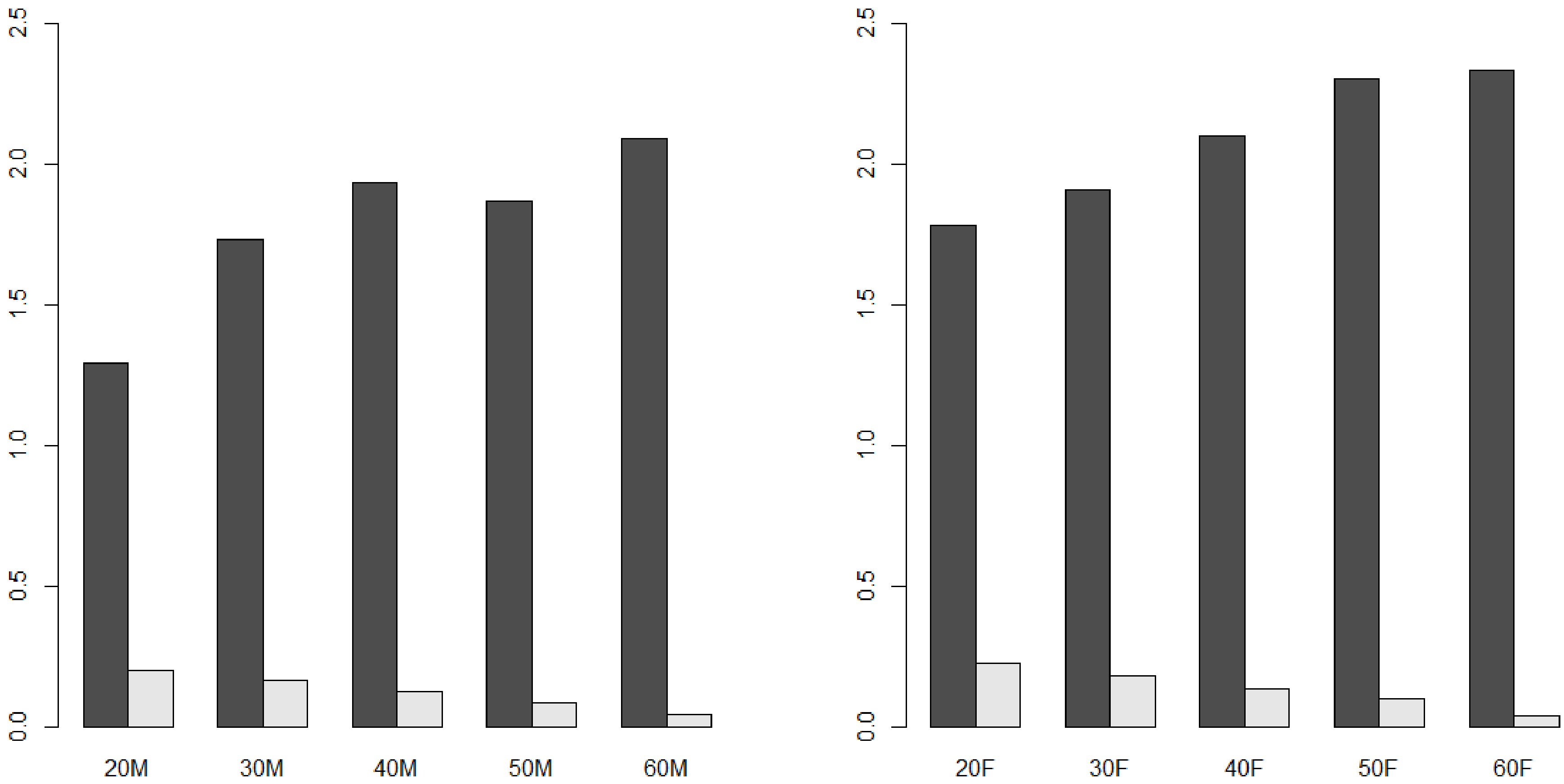
# Segment Cumulative Effect



# Infinite Sum of Cumulative Effect

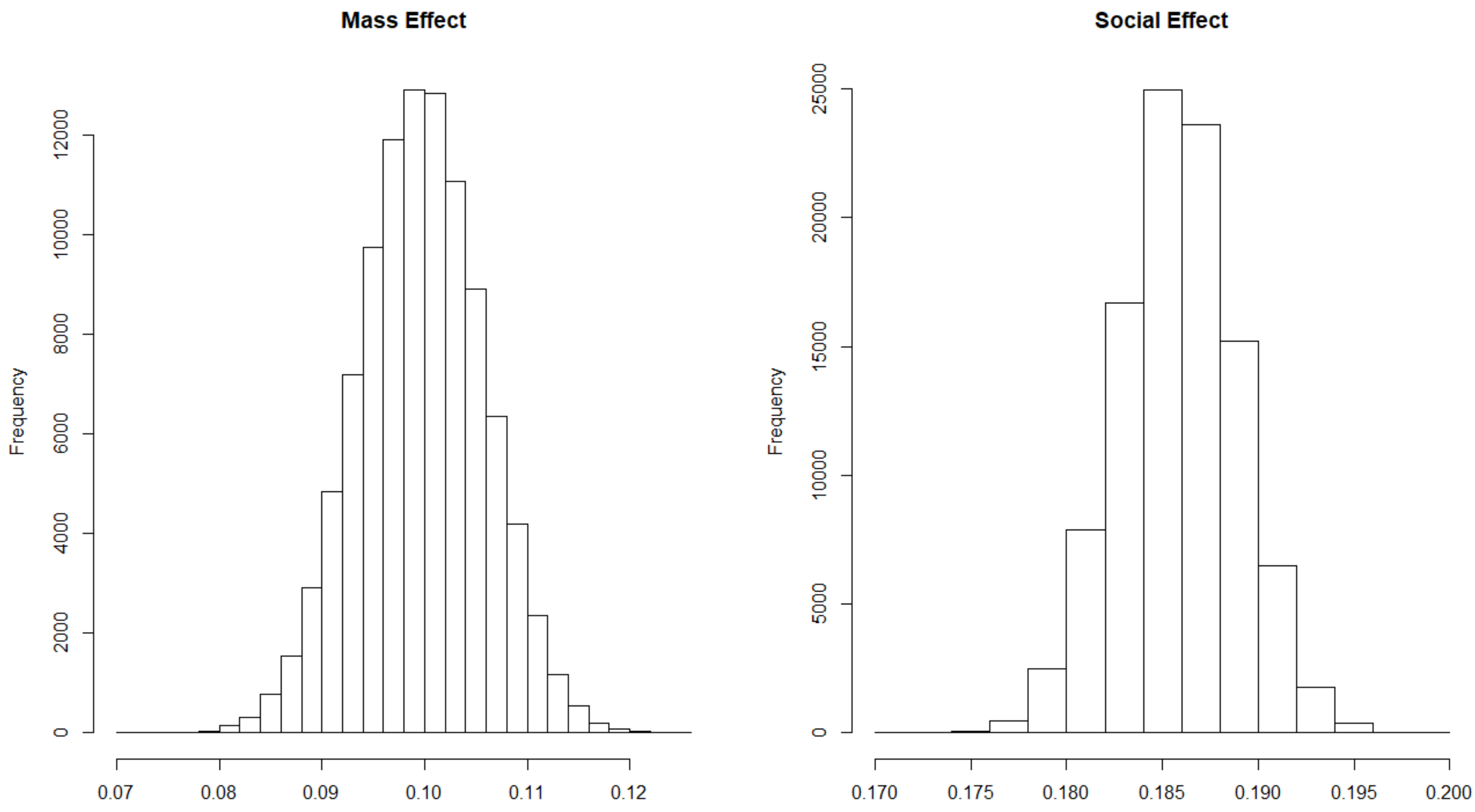
■ : Indirect effect via mass media

□ : Indirect effect via social media



# Individual Heterogeneity

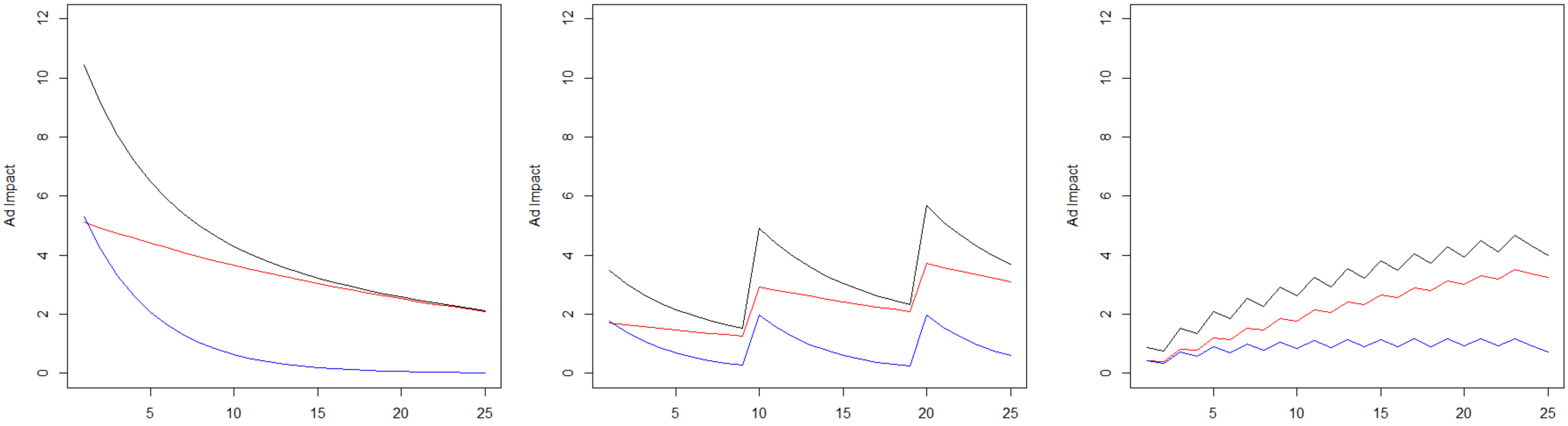
- $b_h \sim N(\hat{\beta}, \hat{\Sigma})$




*N=100000*

# Mass Media Advertising Plans

- For 20s male, 25 weeks, budges=100
  - Left: Expend all (100) in the first week
  - Center: Expend 33.333(=100/3) every 10 weeks
  - Right: Expend 8.3333(=100/12) every two weeks
    - Blue: via Social, Red: via Mass, Black: Total



# Summary of the Results

- Two types of effects of TV advertising
  - **Via Mass effect: indirect effects through mass reach**
  - **Via Social effect: indirect effects through social transmission and reach that is stimulated by mass transmission**
  - Need to incorporate the effect via social media into TV ad ROI
    - **Underestimation of** the effect if only the mass media path is considered
- Advertising and segments
  - The effects are different across segments
    - Younger segments tend to be affected by social, whereas older segments tend to be affected by mass
    - for the same age segment, females are more susceptible than males
- Short and long term effect
  - Effect via mass lasts more than that via social
  - The infinite sum of effects (theoretical values) of mass are far larger than that of social

# Future Issues

- Reconsider the model
  - Examine the precedence of social posts: Model ST incorporates only Mass transmission (MT), other factors need to be considered
  - Need to consider valence of online reviews
    - Negative reviews: Floyed et al. (2014), You et al. (2015), Chevalier and Mayzlin (2006)
- Individual parameters of BLP
  - Σ: Covariance parameters are neither positive nor negative
- Effective ad strategies
  - Based on the result, suggest effective and efficient ad plans

# References

Assael, H. (2011). From silos to synergy: A fifty-year review of cross-media research shows synergy has yet to achieve its full potential. Journal of Advertising Research, 51(1 50th Anniversary Supplement), 42-58.

Balducci, B., & Marinova, D. (2018). Unstructured data in marketing. Journal of the Academy of Marketing Science, 46(4), 557-590.

Berger, J. (2014). Word of mouth and interpersonal communication: A review and directions for future research. Journal of consumer psychology, 24(4), 586-607.

Berger, J., & Milkman, K. L. (2012). What makes online content viral?. Journal of marketing research, 49(2), 192-205.

Berry, S. T. (1994). Estimating discrete-choice models of product differentiation. The RAND Journal of Economics, 242-262.

Berry, S., Levinsohn, J., & Pakes, A. (1995). Automobile prices in market equilibrium. Econometrica: Journal of the Econometric Society, 841-890.

Borah, A., & Tellis, G. J. (2016). Halo (spillover) effects in social media: do product recalls of one brand hurt or help rival brands?. Journal of Marketing Research, 53(2), 143-160.

Chandrasekaran, D., Srinivasan, R., & Sihi, D. (2018). Effects of offline ad content on online brand search: Insights from super bowl advertising. Journal of the Academy of Marketing Science, 46(3), 403-430.

Chevalier, J. A., & Mayzlin, D. (2006). The effect of word of mouth on sales: Online book reviews. Journal of marketing research, 43(3), 345-354.

Dellarocas, C., Zhang, X. M., & Awad, N. F. (2007). Exploring the value of online product reviews in forecasting sales: The case of motion pictures. Journal of Interactive marketing, 21(4), 23-45.

Du, R. Y., Joo, M., & Wilbur, K. C. (2019). Advertising and brand attitudes: Evidence from 575 brands over five years. Quantitative Marketing and Economics, 17(3), 257-323.

Du, R. Y., Xu, L., & Wilbur, K. C. (2019). Immediate Responses of Online Brand Search and Price Search to TV Ads. Journal of Marketing, 83(4), 81-100.

Duan, J. A., & Mela, C. F. (2009). The role of spatial demand on outlet location and pricing. Journal of Marketing Research, 46(2), 260-278.

Dubé, J. P., Hitsch, G. J., & Manchanda, P. (2005). An empirical model of advertising dynamics. Quantitative marketing and economics, 3(2), 107-144.

Fang, E., Li, X., Huang, M., & Palmatier, R. W. (2015). Direct and indirect effects of buyers and sellers on search advertising revenues in business-to-business electronic platforms. Journal of Marketing Research, 52(3), 407-422.

Floyd, K., Freling, R., Alhoqail, S., Cho, H. Y., & Freling, T. (2014). How online product reviews affect retail sales: A meta-analysis. Journal of Retailing, 90(2), 217-232.

Fossen, B. L., & Schweidel, D. A. (2019). Social TV, Advertising, and Sales: Are Social Shows Good for Advertisers?. Marketing Science, 38(2), 274-295.

Jiang, R., Manchanda, P., & Rossi, P. E. (2009). Bayesian analysis of random coefficient logit models using aggregate data. Journal of Econometrics, 149(2), 136-148.

Joo, M., Wilbur, K. C., & Zhu, Y. (2016). Effects of TV advertising on keyword search. International Journal of Research in Marketing, 33(3), 508-523.

Joo, M., Wilbur, K. C., Cowgill, B., & Zhu, Y. (2013). Television advertising and online search. Management Science, 60(1), 56-73.

Joshi, A., & Hanssens, D. M. (2010). The direct and indirect effects of advertising spending on firm value. Journal of marketing, 74(1), 20-33.

Kannan, P. K. & Li, H.A. (2017). Digital marketing: A framework, review and research agenda. International Journal of Research in Marketing, 34(1), 22-45.

Kumar, A., Bezawada, R., Rishika, R., Janakiraman, R., & Kannan, P. K. (2016). From social to sale: The effects of firm-generated content in social media on customer behavior. Journal of Marketing, 80(1), 7-25.

Lamberton, C., & Stephen, A. T. (2016). A thematic exploration of digital, social media, and mobile marketing: Research evolution from 2000 to 2015 and an agenda for future inquiry. Journal of Marketing, 80(6), 146-172.

Liu, Y. (2006). Word of mouth for movies: Its dynamics and impact on box office revenue. Journal of marketing, 70(3), 74-89.

Lodish, L. M., Abraham, M., Kalmenson, S., Livelsberger, J., Lubetkin, B., Richardson, B., & Stevens, M. E. (1995). How TV advertising works: A meta-analysis of 389 real world split cable TV advertising experiments. Journal of Marketing Research, 32(2), 125-139.

Lopez, R. A., Liu, Y., & Zhu, C. (2015). TV advertising spillovers and demand for private labels: the case of carbonated soft drinks. Applied Economics, 47(25), 2563-2576.

Luo, X. (2009). Quantifying the long-term impact of negative word of mouth on cash flows and stock prices. Marketing Science, 28(1), 148-165.

Naik, P. A., & Raman, K. (2003). Understanding the impact of synergy in multimedia communications. Journal of Marketing Research, 40(4), 375-388.

Nevo, A. (2000). Mergers with differentiated products: The case of the ready-to-eat cereal industry. The RAND Journal of Economics, 395-421.

Onishi, H., & Manchanda, P. (2012). Marketing activity, blogging and sales. International Journal of Research in Marketing, 29(3), 221-234.

Stephen, A. T., & Galak, J. (2012). The effects of traditional and social earned media on sales: A study of a microlending marketplace. Journal of marketing research, 49(5), 624-639.

Tellis, G. J., & Franses, P. H. (2006). Optimal data interval for estimating advertising response. Marketing Science, 25(3), 217-229.

Tirunillai, S., & Tellis, G. J. (2014). Mining marketing meaning from online chatter: Strategic brand analysis of big data using latent dirichlet allocation. Journal of Marketing Research, 51(4), 463-479.

Train, K. E. (2009). Discrete choice methods with simulation. Cambridge university press.

Vakratsas, D., & Ma, Z. (2005). A look at the long-run effectiveness of multimedia advertising and its implications for budget allocation decisions. Journal of Advertising Research, 45(2), 241-254.

Wedel, M., & Kannan, P. K. (2016). Marketing analytics for data-rich environments. Journal of Marketing, 80(6), 97-121.

You, Y., Vadakkepatt, G. G., & Joshi, A. M. (2015). A meta-analysis of electronic word-of-mouth elasticity. Journal of Marketing, 79(2), 19-39.

Yuan, Y., & MacKinnon, D. P. (2009). Bayesian mediation analysis. Psychological methods, 14(4), 301.

大阪大学
OSAKA UNIVERSITY

# Appendix: Utility and Demand Function

# Utility and Choice Probability

- $U_{ijt}$: utility for brand $j$ $(j = 0, \cdots, J)$ at time $t$ by consumer $i$,
$$U_{ijt} = \mu_{ijt} + e_{ijt}$$
- Let the utility of outside goods $\mu_{i0t}$ be $0$,
$$U_{i0t} = 0 + e_{i0t}$$
- Assume that $e_{ijt}$ follows the *Type I extreme value distribution*, choice probability is obtained as follows (e.g., Train 2012),
$$s_{ijt} = \frac{\exp(\mu_{ijt})}{1 + \sum_{k=1}^{J} \exp(\mu_{ijt})}.$$

# Choice Probability

- Assume that component of $\mu_{ijt}$ is a random coefficient linear model,
$$\mu_{ijt} = X_{jt}\theta_i + \eta_{jt}$$
- Let $\theta_i = \beta + v_i, v_i \sim N(0, \Sigma)$ and $\eta_{jt} \sim N(0, \tau^2)$, the choice probability is as follows:
$$p_{ijt} = \frac{\exp\left(X_{jt}\theta_i + \eta_{jt}\right)}{1 + \sum_{k=1}^{J} \exp\left(X_{jt}\theta_i + \eta_{jt}\right)}$$
- Rewrite the above equation using $\beta$ and $v_i$,
$$p_{ijt} = \frac{\exp\left(X_{jt}(\beta + v_i) + \eta_{jt}\right)}{1 + \sum_{k=1}^{J} \exp\left(X_{jt}(\beta + v_i) + \eta_{jt}\right)}$$
- Note that $E(v_i) = 0$, and $E\left(X_{jt}v_i\right) = 0$.

# Aggregating Choice Probabilities

- Let $E = \{\mu_{ijt} | u_{ijt} \geq u_{ikt}, k = 1, \cdots, J\}$, and distribution of the consumer heterogeneity as $P(E)$, the market share is theoretically obtained as follows (Berry et al. 1995):

$$s_{jt} = \int_E P(E)$$

- The simplest way to estimate the parameters, we can assume that market share is equivalent to the finite sum of the choice probabilities of consumers (Berry 1994),

$$s_{jt} = \frac{1}{N} \sum_{i=1}^{N} \frac{\exp(\mu_{ijt})}{1 + \sum_{k=1}^{J} \exp(\mu_{ijt})}$$

- Obtain the geometric mean

$$\log s_{jt} = \frac{1}{N} \sum_{i=1}^{N} \log \frac{\exp(\mu_{ijt})}{1 + \sum_{k=1}^{J} \exp(\mu_{ijt})}$$

$$\log s_{jt} = \frac{1}{N} \sum_{i=1}^{N} \mu_{ijt} - \log\left(1 + \sum_{k=1}^{J} \exp(\mu_{ijt})\right)$$

- Subtract the probability of the outside goods from both side

$$\log s_{jt} - \log s_{0t} = \frac{1}{N} \sum_{i=1}^{N} \mu_{ijt}$$

# Aggregating Choice Probabilities

- The finite sum of the choice probability

$$\log s_{jt} - \log s_{0t} = \frac{1}{N}\sum_{i=1}^{N}\mu_{ijt}$$

  - where,

$$\frac{1}{N}\sum_{i=1}^{N}\mu_{ijt} = \frac{1}{N}\sum_{i=1}^{N}\left[X_{jt}(\beta + v_i) + \eta_{jt}\right]$$

- Therefore, we get the following equation:

$$\log s_{jt} - \log s_{0t} = X_{jt}\beta + \eta_{jt} + \frac{1}{N}\sum_{i=1}^{N}X_{jt}v_i$$

- For large $N$, we can ignore $\frac{1}{N}\sum_{i=1}^{N}X_{jt}v_i \approx 0$, and obtain the following linear regression model:

$$\log s_{jt} - \log s_{0t} = X_{jt}\beta + \eta_{jt}$$

# Aggregating Choice Probabilities

- Aggregate demand

$$\log s_{jt} - \log s_{0t} = X_{jt}\beta + \eta_{jt}, \eta_{jt} \sim N(0, \tau^2)$$

- Estimate parameters using aggregate market variables
  - $s_{jt}$: Market share
    - $s_{0t}$: Market share of outside goods
  - $X_{jt}$: Marketing variables
  - $\varepsilon_{jt}$: Normal error term
- However, we cannot obtain variance of consumers from this model

# Aggregate Demand Function

- Choice probability

$$p_{ijt} = \frac{\exp\left(X_{jt}(\beta + v_i) + \eta_{jt}\right)}{1 + \sum_{k=1}^{J} \exp\left(X_{jt}(\beta + v_i) + \eta_{jt}\right)}$$

- Since individual variation term is $v_i$, we can obtain the aggregate demand function introducing its density $\pi(v_i|\Sigma)$.
  - Jiang, Manchanda, and Rossi (2009)
- Market share

$$s_{jt} = \int \left[\frac{\exp\left(X_{jt}(\beta + v_i) + \eta_{jt}\right)}{1 + \sum_{k=1}^{J} \exp\left(X_{jt}(\beta + v_i) + \eta_{jt}\right)}\right] \pi(v_i|\Sigma)\, dv_i$$

# Aggregate Demand Function

- Let market share is equivalent to function $h$,

$$s_{jt} = h\left(\eta_{jt} \middle| X_t, \beta, \Sigma\right)$$

  - $h$ is function of $\eta_{jt}$ given by parameters $\{X_t, \beta, \Sigma\}$

- Since $\eta_{jt} \sim N(0, \tau^2)$, we can obtain the following inverse function:

$$\eta_t = h^{-1}(s_t|X_t, \beta, \Sigma) \sim N_J\left(0, \mathrm{diag}(\tau^2)\right)$$

- Densify of function $h^{-1}$: Normal density

$$\pi(h^{-1}(s_t|X_t, \beta, \Sigma)) = \phi(\eta_t|\tau^2)$$

- We can rewrite the above function as follows:

$$\pi(s_t) = \phi(h^{-1}(s_t|X_t, \beta, \Sigma)|\tau^2)\, J_{\eta_t \to s_t}$$

$$\Leftrightarrow \pi(s_t) = \phi(h^{-1}(s_t|X_t, \beta, \Sigma)|\tau^2)\left(J_{s_t \to \eta_t}\right)^{-1}$$

# Likelihood Function

- Density of $s_t$ is as follows:

$$\pi(s_t|\beta,\Sigma,\tau^2) = \phi(h^{-1}(s_t|X_t,\beta,\Sigma)|\tau^2)\left(J_{s_t\to\eta_t}\right)^{-1}$$

- Based on the above function, the likelihood function is defined as follows:

$$L(\beta,\Sigma,\tau^2) = \prod_{t=1}^{T} \pi(s_t|\beta,\Sigma,\tau^2)$$

- Full conditional posterior distribution is as follows:

$$\pi(\beta,\Sigma,\tau^2|D) \propto L(D|\beta,\Sigma,\tau^2)\pi(\beta)\pi(\Sigma)\pi(\tau)$$

# Prior Distribution

- Prior distribution

$$\beta \sim N_K(m_{0\theta}, V_{0\theta})$$
$$\tau^{-2} \sim Ga(g_0/2, G_0/2)$$

- Prior of $\Sigma$
  - Assume prior parameters for $R$ which is the Cholesky root of $\Sigma$ ($RR' = \Sigma$)
  - Diagonal elements: $r_{ii} = \exp(d_i)$, $d_i \sim N(0, s_d^2)$
  - Off-diagonal elements: $r_{ij} \sim N(0, s_d^2)$
  - $R = \begin{pmatrix} r_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ r_{K1} & \cdots & r_{KK} \end{pmatrix}$

# Numerical Approximation

- Market Share

$$s_{jt} = \int \left( \frac{\exp(m_{itj})}{1 + \sum_{k=1}^{J} \exp(m_{itk})} \right) \phi(v_i|\Sigma)\, dv$$

- where, $m_{itj} = X_{jt}\bar{\theta} + \eta_{jt} + X_{jt}v_i, v_i \sim N(0, \Sigma)$
- Numerical approximation
  - Step 0) Let $\mu_{jt}^{O} = X_{jt}\beta + \eta_{jt}$.
  - Step 1) Generate $v^{(h)} \sim N(0, \Sigma), h = 1, \cdots, H$.
  - Step 2) Evaluate $m_{tj}^{(h)} = \mu_{jt}^{O} + X_{jt}v^{(h)}$ and $s_{tj}^{(h)} = \left( \frac{\exp\left(m_{tj}^{(h)}\right)}{1+\sum_{k=1}^{J} \exp\left(m_{tk}^{(h)}\right)} \right)$.
  - Step 3) Obtain $\hat{s}_{jt} = \frac{1}{H}\sum_{h=1}^{H} s_{tj}^{(h)}$.
  - Step 4) Obtain $\mu_{jt}^{N} = \mu_{jt}^{O} + \log(s_{jt}) - \log(\hat{s}_{jt})$.
  - Step 5) If $\max_{(j,t)}\left( \left| \frac{\mu_{jt}^{N} - \mu_{jt}^{O}}{\mu_{jt}^{N}} \right| \right) < c$, Let $\mu_{jt} = \mu_{jt}^{N}$, otherwise, let $\mu_{jt}^{O} \leftarrow \mu_{jt}^{N}$ and return to Step 2
  - As a result, we can obtain numerical approximations of $v^{(h)}$, $s_{jt}^{(h)}$, $\mu_{jt}$, $\hat{s}_{jt}$, and $\eta_{jt}$.

# Jacobian

- Jacobian

$$J_{s_t \to \eta_t} = \begin{vmatrix} \frac{\partial s_{1t}}{\partial \eta_{1t}} & \cdots & \frac{\partial s_{1t}}{\partial \eta_{Jt}} \\ \vdots & \ddots & \vdots \\ \frac{\partial s_{Jt}}{\partial \eta_{1t}} & \cdots & \frac{\partial s_{Jt}}{\partial \eta_{Jt}} \end{vmatrix}$$

$$\frac{\partial s_{jt}}{\partial \eta_{kt}} = \int -s_{ijt} s_{ikt} \phi(v_i|\Sigma)\, d\theta^i \text{ , if } k \neq j$$

$$\frac{\partial s_{jt}}{\partial \eta_{kt}} = \int s_{ijt}(1 - s_{ijt}) \phi(v_i|\Sigma)\, d\theta^i \text{ , if } k = j$$

- Numerical approximation: $s_{tj}^{(h)}$

$$\frac{\partial s_{jt}}{\partial \eta_{kt}} \approx \sum_{h=1}^{H} -s_{jt}^{(h)} s_{ikt}^{(h)} \text{ , if } k \neq j$$

$$\frac{\partial s_{jt}}{\partial \eta_{kt}} \approx \sum_{h=1}^{H} \left(1 - s_{jt}^{(h)}\right) s_{ikt}^{(h)} \text{ , if } k = j$$

# Evaluating Likelihood Function

- $v^{(h)}, h = 1, \cdots, H$ are available from numerical approximation
- Evaluate likelihood function from $v^{(h)}$,

$$L(\beta, \Sigma, \tau^2) = \prod_{t=1}^{T} \pi(s_t|\beta, \Sigma, \tau^2)$$

- Components of the likelihood function

$$\pi(s_t|\beta, \Sigma, \tau^2) = \phi(\eta_t|\tau^2)\left(J_{s_t \to \eta_t}\right)^{-1}$$

  - Since $\eta_t \sim N(0, \tau^2 I_K)$, density is available using $\eta_t$
  - Using $s_{jt}^{(h)}$, we can evaluate Jacobian

# Sampling Σ

- Collect samples from random walk M-H method
- Generation of candidate sample
  - Diagonal elements of the previous sample: $r_{ii}^{O} = \exp\left(d_i^{O}\right)$
  - Off-diagonal elements of the previous sample: $r_{ij}^{O}$
  - Diagonal elements of candidate sample: $r_{ii}^{N} = \exp\left(d_i^{N}\right), d_i^{N} \sim N\left(d_i^{O}, s_d^2\right)$
  - Off-diagonal elements of candidate sample: $r_{ij}^{N} \sim N\left(r_{ij}^{O}, s_d^2\right)$
- Obtain $\Sigma^N$ from the above candidate samples
- Acceptance rate

$$a = \max\left\{1, \frac{L(\beta, \Sigma^N, \tau^2)\pi(\Sigma^N)}{L(\beta, \Sigma^O, \tau^2)\pi(\Sigma^O)}\right\}$$

# Sampling $\beta$ and $\tau^2$

- Likelihood function:

$$L(D|\beta,\Sigma,\tau^2) = \prod_{t=1}^{T} \pi(s_t|\beta,\Sigma,\tau^2) = \prod_{t=1}^{T} \phi(\eta_t|\tau^2)\left(J_{s_t \to \eta_t}\right)^{-1}$$

Note that Jacobian $J_{s_t \to \eta_t}$is independent from $\beta$ and $\tau^2$, therefore we do not need to consider $J_{s_t \to \eta_t}$ to collect samples of the posterior of $\beta$ and $\tau^2$

- Conditional posterior of $\beta$:

$$\beta|\Sigma,\tau^2,D \sim N(m_1,V_{1\theta})$$

$$V_{1\theta} = \left(\tau^{-2}X'X + V_{0\theta}^{-1}\right)^{-1}, m_{1\theta} = V_{1\theta}\left(\tau^{-2}X'\mu + V_{0\theta}^{-1}m_{0\theta}\right)$$

- Conditional Posterior of $\tau^2$:

$$\tau^{-2}|\Sigma,\beta,D \sim Ga(g_1/2, G_1/2)$$

$$g_1 = g_0 + TJ, G_1 = \left(G_0^{-1} + \sum_{t=1}^{T}\sum_{J=1}^{J}\eta_{jt}^2\right)^{-1}$$