Capturing Substitutions using Flexible Utility Function for Outside Good

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- **Introduction**
- **Model**
- **Empirical analysis**
- **Concluding remarks**

Introduction

- ✓ **Pricing decision**
- ✓ **Substitution**
- ✓ **Multiple discreteness**

$$\left(x_1, x_2, \dots, x_J\right)$$
$$\left(p_1, p_2, \dots, p_J\right)$$

- ✓ **Outside good**

$$\left(x_1, x_2, \dots, x_J, {\color{red}z}\right)$$

- ✓ Price effect
- ✓ Elasticity

Introduction

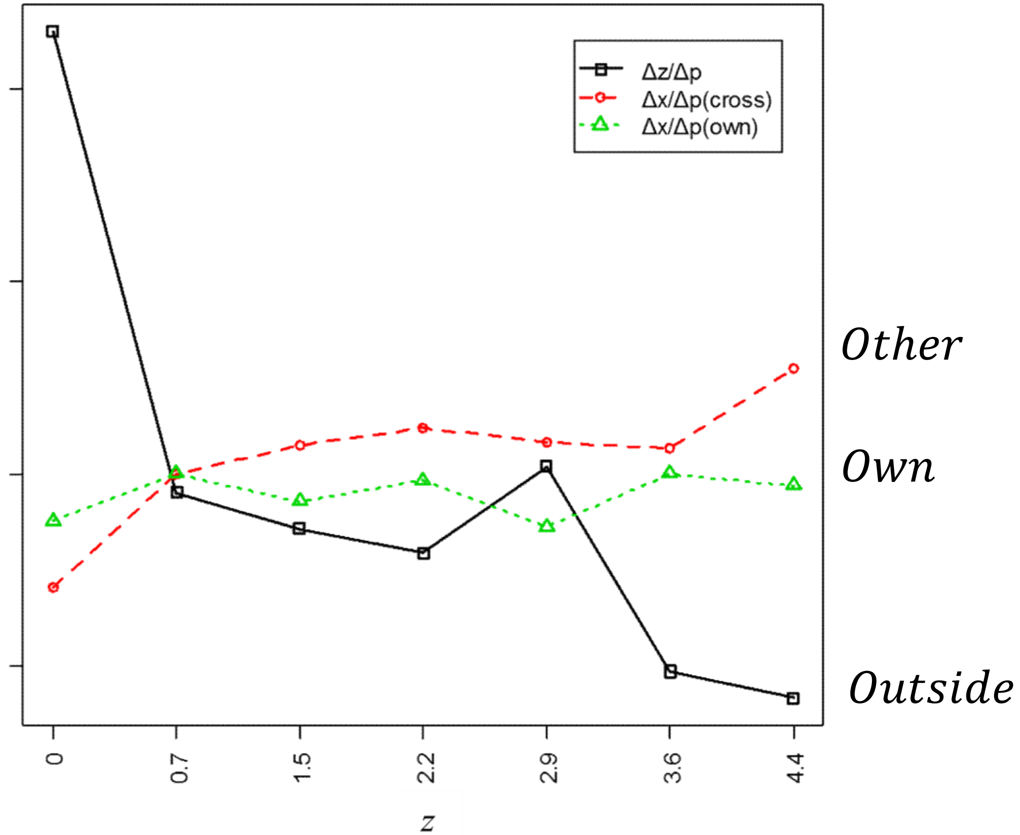
- **Substitution** among product offerings occurring through *price responses*

- In direct utility model, price change by one good affects demand on
  - other goods
  - outside good

- Often, the price effects are **dependent** on the level of **outside good**

Evidence from Data (Potato chip)

IRI panel dataset (Bronnenberg et al. 2008)



The demand changes in response to a price increase of a good k (p_k)

Introduction (cont'd)

- Demand (x, z) from constrained maximization:

$$\max. U(x,z) = \sum_{j=1}^{J} u_x(x_j) + u_z(z)$$

$$s.t. \sum_{j=1}^{J} p_j x_j + z \leq M$$

$$\frac{\partial z}{\partial p_k}, \quad \frac{\partial x_k}{\partial p_k}, \quad \frac{\partial x_j}{\partial p_k}$$

→ Substitution to be inferred accounting for the **interaction** with the level of **outside good**(z)

Introduction (cont'd)

- **How?** $u_z(z)$
  - Satiation ! (a.1)
  - Flexible enough (a.2)

- Literature in direct utility model is silent about this issue
  - $u_z(z)$ is mostly standard log, $ln(z)$
  - while (a.1) is satisfied, (a.2) is not.

  $\rightarrow$ Price effect$\left(\frac{\partial x_j}{\partial p_k}, \frac{\partial x_k}{\partial p_k}, \frac{\partial z}{\partial p_k}\right)$ not dependent on z

  $\rightarrow$ Elasticity estimates in question

Goal

- To propose a direct utility model with proper specification for $u_z(z)$ that allows for the dependence of price effect on the outside good
  - Flexible substitution
  - Proper elasticity-based pricing

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- Conclusion

Literature on DUM: *structure*

- Direct utility function

$$U(\boldsymbol{x}_t, z_t) = \sum_{j=1}^{J} u_x(x_{jt}) + u_z(z_t)$$

$$s.t. \sum_{j=1}^{J} p_{jt}x_{jt} + z_t \leq M_t$$

- K-T condition

$$\varepsilon_{jt} = g_{jt} \text{ if } x_{jt} > 0$$

$$\varepsilon_{jt} < g_{jt} \text{ if } x_{jt} = 0$$

$$g_{jt} = -\ln\left(\frac{\partial u_x(x_{jt})}{\partial x_{jt}}\right) + \ln\left(p_{jt} \cdot \frac{\partial u_z(z_t)}{\partial z_t}\right)$$

Literature on DUM: *inference*

- Likelihood

$$L(\boldsymbol{x}_t, z_t) = \prod_{\{x_{jt}>0\}} f_\varepsilon(g_{jt}) \prod_{\{x_{jt}=0\}} F_\varepsilon(g_{jt}) \cdot |J_t|$$

where

$$J_{t(i,j)} = \frac{\partial g_{it}}{\partial x_{jt}}$$

→ Note : $|J_t| = \left| J_t\left(\frac{u''(z_t)}{u'(z_t)}\right) \right|$

Substitution pattern

$$U(\boldsymbol{x}, z) = \sum_j \frac{\psi_j}{\gamma_j} \ln\left(\gamma_j x_j + 1\right) + u(z)$$

- $m_{zk}(z) \equiv \frac{\partial z}{\partial p_k} = \frac{1}{\gamma_k} \cdot \frac{1}{1+h(z)\sum_{l\in R} \psi_l/\gamma_l}$
- $m_{jk}(z) \equiv \frac{\partial x_j}{\partial p_k} = \frac{1}{p_j \gamma_k} \cdot \frac{h(z)\psi_l/\gamma_l}{1+h(z)\sum_{l\in R} \psi_l/\gamma_l}$
- $m_{kk}(z) \equiv \frac{\partial x_k}{\partial p_k} = -\frac{1}{p_k \gamma_k} \cdot \frac{1+h(z)\sum_{l\in R, l\neq k} \psi_l/\gamma_l}{1+h(z)\sum_{l\in R} \psi_l/\gamma_l} - \frac{x_k}{p_k}$

where $h(z) = \frac{-u''(z)}{u'(z)^2}$

Condition required $h'(z) > 0$

$$\left.\begin{array}{l} \dfrac{\partial m_{zk}(z)}{\partial z} < 0 \\[2em] \dfrac{\partial m_{jk}(z)}{\partial z} > 0 \\[2em] \dfrac{\partial m_{kk}(z)}{\partial z} > 0 \end{array}\right\} \quad iff \quad h'(z) > 0$$

Summary : $U_z(z)$ in literature

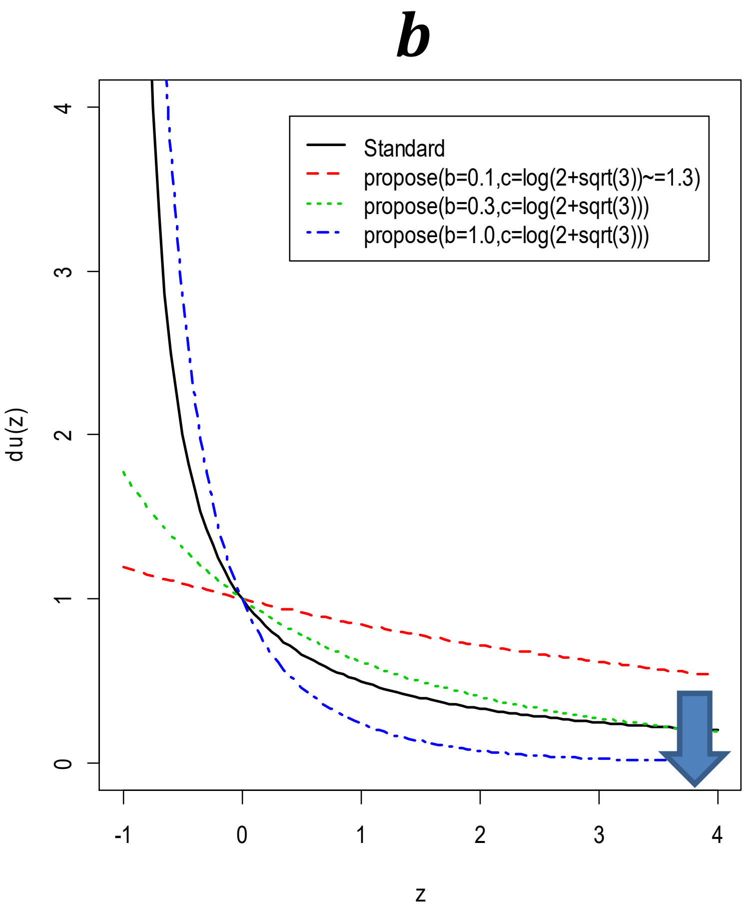
U(z)	Satiation	$h(z)$	
Log function[1]	*yes*	**Constant**	$h'(z) = 1$
This study	*yes*	Flexible	$h'(z) > 0$ guaranteed

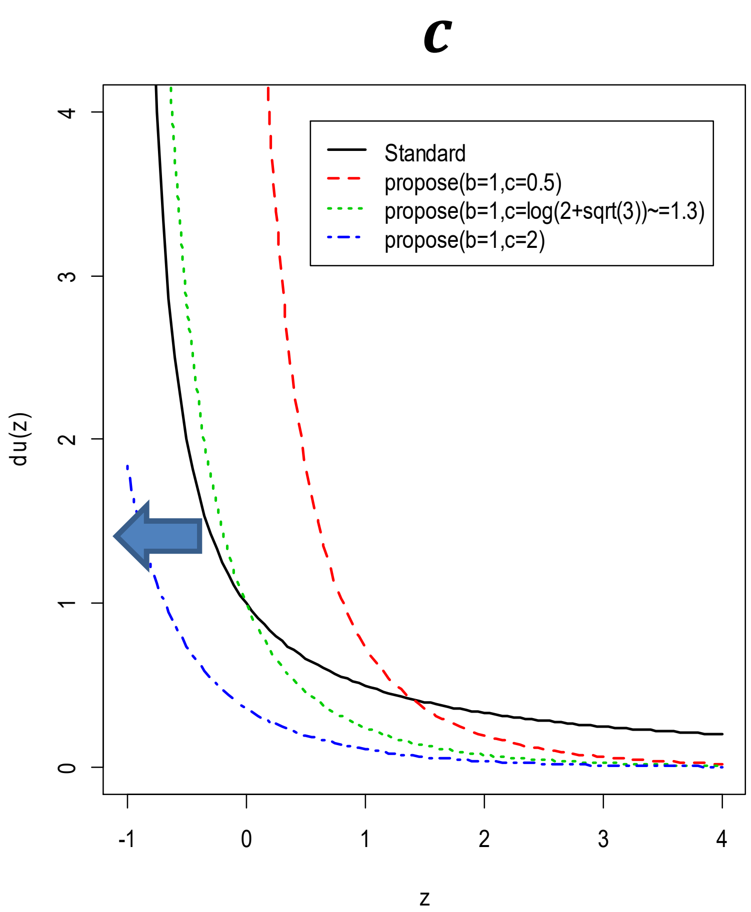
[1]Satomura et al(2011), Hasegawa et al(2012), Lee et al(2013), Howell et al(2016), Kang et al(2016), Kim et al(2017)

cf. Kim et al(2002, 2007), Bhat(2008), Luo et al(2012),

$U_z(z)$

$$u(z) = \frac{1}{b} \cdot \frac{1 + exp(bz + c)}{1 - exp(bz + c)}$$





$U_z(z)$

$$u(z) = \frac{1}{b} \cdot \frac{1 + exp(bz + c)}{1 - exp(bz + c)}$$

$$\rightarrow \quad u'(z) = \frac{2 \cdot exp(bz + c)}{\left(1 - exp(bz + c)\right)^2}$$

$$\rightarrow \quad h(z) = b \cdot sinh(bz + c)$$

$$h'(z) = b^2 \cdot cosh(bz + c) \; > 0$$

- Introduction
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- **Empirical Analysis**
- Concluding remarks

Data

	LAYS	WAVY LAYS	RUFFLES	OLD DUTCH
Average Price ($)	2.21	2.38	2.67	0.87
Total Quantity	4,598	2,447	1,072	755
Total Incidence	3,246	1,869	839	591
1 unit	2,147	1,397	626	446
2 units	899	390	196	131
more than 2 units	200	82	17	14

including UPCs with m/s > 1% and size being medium (12 – 16 oz).

- 476 households with at least 3 purchase occasions
- Total 5,941 purchase occasions,
- On average, 1.49 units purchased per trip.

Estimation

$$U(\mathbf{x}, z) = \sum_j \frac{\psi_{h,j}}{\gamma_{h,j}} \ln\left(\gamma_{h,j} x_j + 1\right) + \frac{1}{b_h} \cdot \frac{1 + exp(b_h z + c_h)}{1 - exp(b_h z + c_h)}$$

- Heterogeneity

$$\boldsymbol{\theta}_h = \left(\tilde{\psi}_{1,h}, \dots, \tilde{\psi}_{4,h}, \tilde{\gamma}_{1,h}, \dots, \tilde{\gamma}_{4,h}\right)' \sim N(\bar{\boldsymbol{\theta}}, \mathbf{V}_\theta)$$

$$\boldsymbol{\alpha}_h = \left(\tilde{b}_h, \tilde{c}_h\right)' \sim N(\bar{\boldsymbol{\alpha}}, \mathbf{V}_\alpha)$$

where

$$\tilde{\psi}_h = ln(\psi_h),\ \tilde{\gamma}_h = ln(\gamma_h),\ \tilde{b}_h = ln(b_h),\ \tilde{c}_h = ln(c_h)$$

Estimation

$$U(\mathbf{x}, z) = \sum_j \frac{\psi_{h,j}}{\gamma_{h,j}} \ln\left(\gamma_{h,j} x_j + 1\right) + \frac{1}{b_h} \cdot \frac{1 + exp(b_h z + c_h)}{1 - exp(b_h z + c_h)}$$

- MCMC

$$\{\boldsymbol{\theta}_h, h = 1, .., H\} \middle| \ \bar{\boldsymbol{\theta}}, \mathbf{V}_\theta, \{\mathbf{x}_h, \mathbf{p}_h\}$$

$$\bar{\boldsymbol{\theta}} \ \middle| \{\boldsymbol{\theta}_h, h = 1, .., H\}, \mathbf{V}_\theta, \{\mathbf{x}_h, \mathbf{p}_h\}$$

$$\mathbf{V}_\theta \ \middle| \{\boldsymbol{\theta}_h, h = 1, .., H\}, \bar{\boldsymbol{\theta}}, \{\mathbf{x}_h, \mathbf{p}_h\}$$

$$\{\boldsymbol{\alpha}_h, h = 1, .., H\} \middle| \ \bar{\boldsymbol{\alpha}}, \mathbf{V}_\alpha, \{\mathbf{x}_h, \mathbf{p}_h\}$$

$$\bar{\boldsymbol{\alpha}} \middle| \{\boldsymbol{\alpha}_h, h = 1, .., H\}, \mathbf{V}_\alpha, \{\mathbf{x}_h, \mathbf{p}_h\}$$

$$\mathbf{V}_\alpha \middle| \{\boldsymbol{\alpha}_h, h = 1, .., H\}, \bar{\boldsymbol{\alpha}}, \{\mathbf{x}_h, \mathbf{p}_h\}$$

Model Fit

Model Fit		Benchmark Model	Proposed Model
In-sample	LMD	-11,581	-11,015
	SSE	15,645	7,237
Predictive	SSE	17,020	15,062

$$U(\mathbf{x}, z) = \sum_j \frac{\psi_{h,j}}{\gamma_{h,j}} \ln\left(\gamma_{h,j} x_j + 1\right) + \psi_z \cdot log(z)$$

$$U(\mathbf{x}, z) = \sum_j \frac{\psi_{h,j}}{\gamma_{h,j}} \ln\left(\gamma_{h,j} x_j + 1\right) + \frac{1}{b_h} \cdot \frac{1 + exp(b_h z + c_h)}{1 - exp(b_h z + c_h)}$$

Posterior distribution

Parameter	Benchmark Model			Proposed Model		
	Mean	*Std.*	Hetero	Mean	*Std.*	Hetero
$\bar{\tilde{\psi}}_1$	set to 0	-		set to 0	-	
$\bar{\tilde{\psi}}_2$	-0.65	*0.09*	2.45	-0.56	*0.07*	1.54
$\bar{\tilde{\psi}}_3$	-1.47	*0.09*	2.24	-1.36	*0.08*	1.28
$\bar{\tilde{\psi}}_4$	-3.10	*0.12*	4.00	-3.00	*0.10*	2.64
$\bar{\tilde{\gamma}}_1$	-0.71	*0.07*	1.04	-1.10	*0.06*	0.46
$\bar{\tilde{\gamma}}_2$	-1.48	*0.11*	1.11	-1.71	*0.14*	1.26
$\bar{\tilde{\gamma}}_3$	-4.05	*0.28*	2.58	-4.46	*0.46*	2.20
$\bar{\tilde{\gamma}}_4$	-1.50	*0.15*	1.11	-1.66	*0.13*	1.18
$\bar{\tilde{\psi}}_z$	0.53	*0.05*	0.90			
$\bar{\tilde{b}}$				-1.21	*0.04*	0.50
$\bar{\tilde{c}}$				-0.12	*0.02*	0.11

Price Elasticity $(\eta_{j \leftarrow k})$

Benchmark model

$$u(z) = \psi_z \cdot log(z)$$

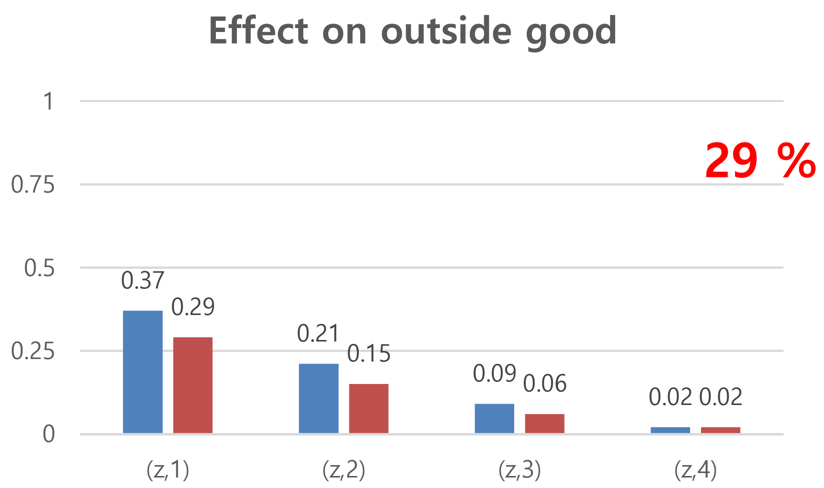
	1	2	3	4	z
1	-1.20	.30	.10	.02	.37
2	.61	-1.63	.14	.02	.21
3	.48	.41	-2.01	.00	.09
4	.26	.18	.06	-1.53	.02

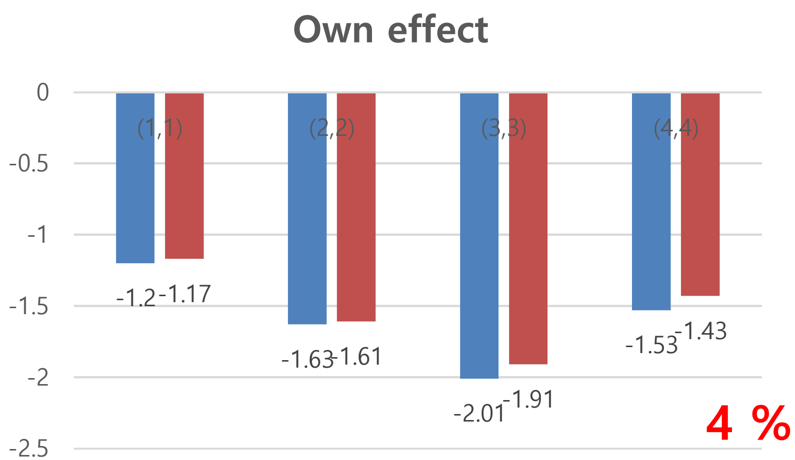
Proposed model

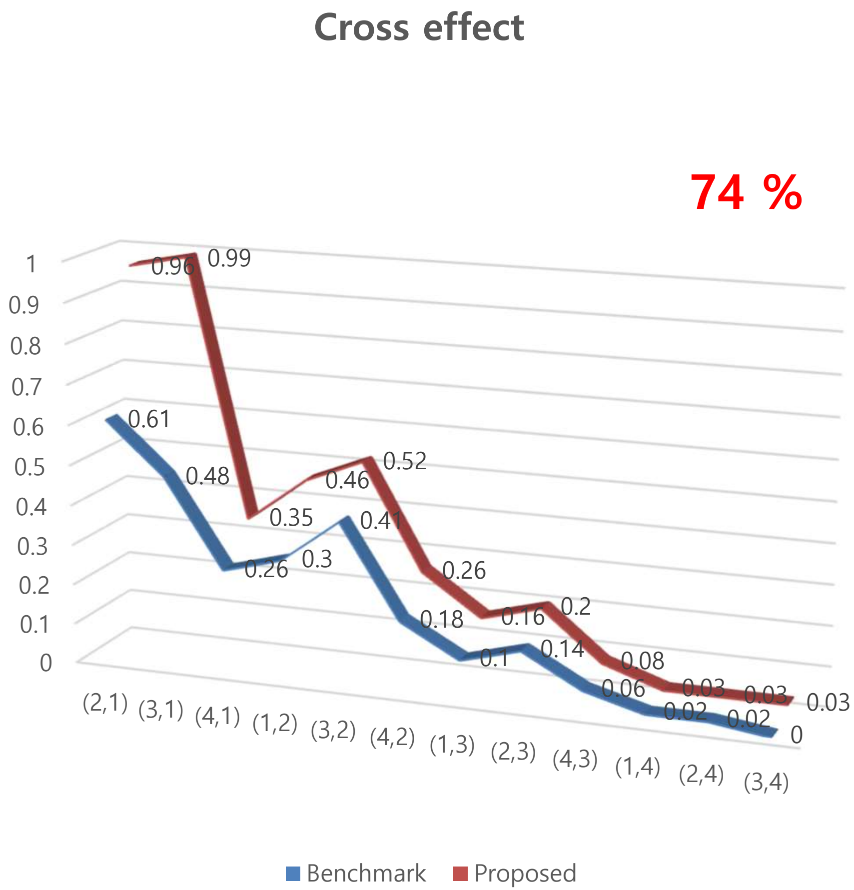
$$u(z) = \frac{1}{b} \cdot \frac{1 + exp(bz + c)}{1 - exp(bz + c)}$$

	1	2	3	4	z
1	-1.17	.46	.16	.03	.29
2	.96	-1.61	.20	.03	.15
3	.99	.52	-1.91	.03	.06
4	.35	.26	.08	-1.43	.02

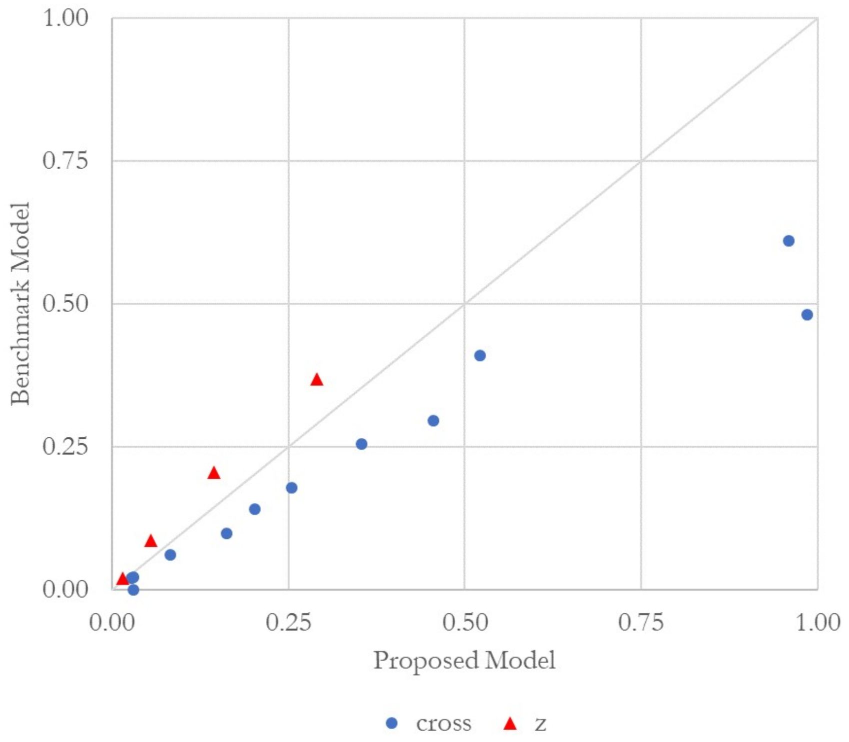
Price Elasticity







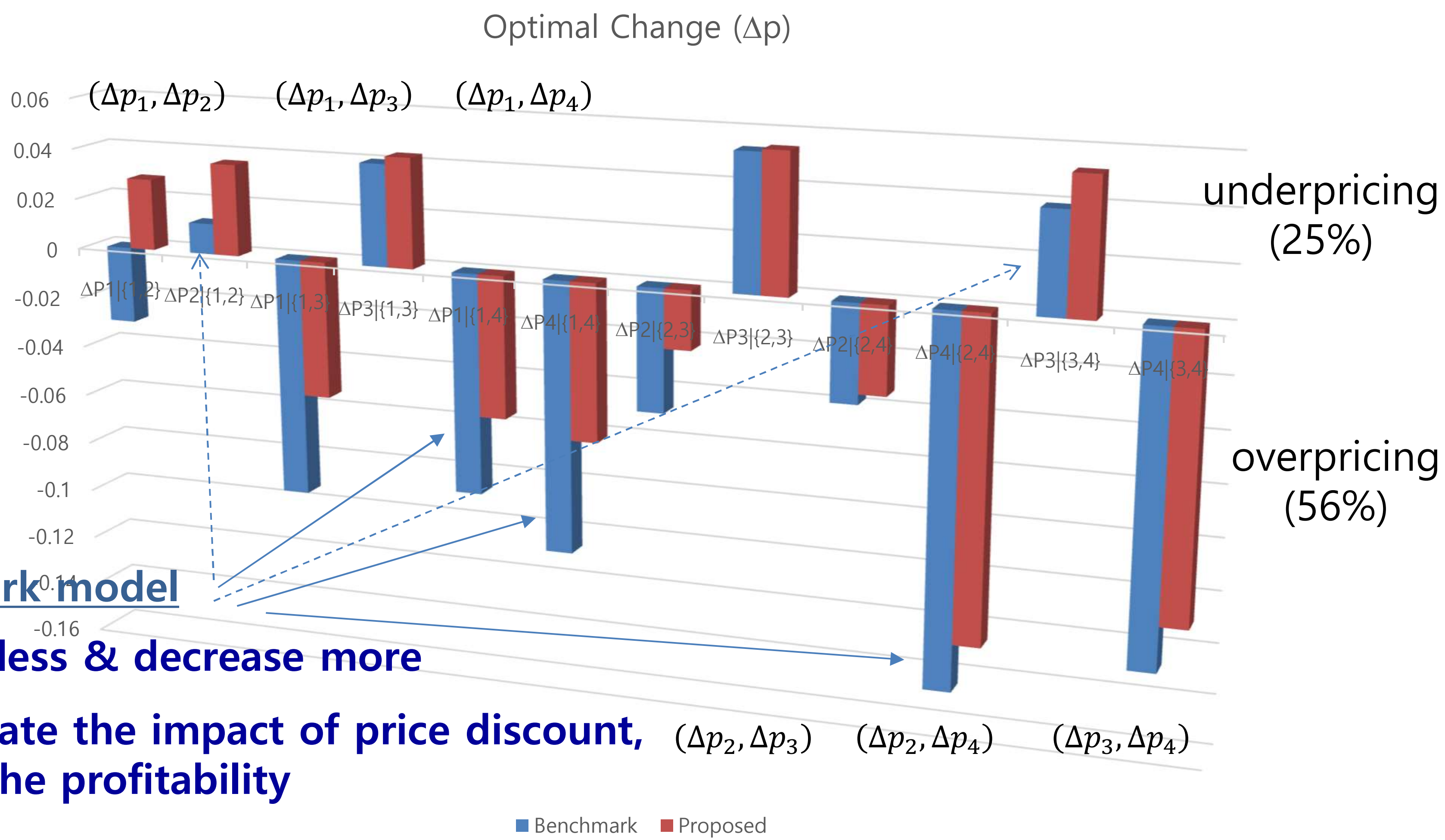
Price Elasticity



Counterfactual : Line pricing

- Changes in price**s** for **multiple** products : manufacturers or retailers to price for a targeted 'group' of offerings (revenue maximization)
  - 4 product offerings :$(x_1, x_2, x_3, x_4), (p_1, p_2, p_3, p_4)$
  - Changing prices for **two** of them simultaneously
  - Optimal change: $\left(\Delta p_i, \Delta p_j\right) \middle| \left(p_i, p_j, \check{\mathbf{p}}_{k \neq i,j}\right)$

Optimal price change



Benchmark model

Increase less & decrease more

Overly state the impact of price discount, hurting the profitability

Counterfactual : Line pricing

- Whether/How much to change (Δp)
  - increase p : own effect (-) vs cross effect (+ vs -)
  - decrease p : own effect (+) vs cross effect (- vs +)

Proposed < Benchmark

Proposed > Benchmark

→ dependent on ***relative*** difference in cross effect between goods 'in' and 'outside' the product line

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Concluding remarks

- **Demand response to price change being dependent on the level of outside good**
- **An approach to capture the flexible substitution via new utility function of outside good**
  - more than satiation (flexibility)
- **Multi-product pricing**
  - Proper cross-elasticity estimates

Concluding remarks

- **More on consumer's constrained u.max decision**
  - Utility structure : preference, satiation
  - Constraint structure: single to multi-, linear to nonlinear
  - Incorporating 'partition': consumer preferences within each group can be described independently of consumption levels in other groups (for larger assortment)

감사합니다

Thank you

ありがとうございました