

Impact of Technology Development Costs on Licensing Form in a Differentiated Cournot Duopoly

Tatsuya Kitagawa^a, Yasushi Masuda^b, Masashi Umezawa^{c 1 2}

^aGE Japan Corporation, Minato-ku, Tokyo, JAPAN

^bFaculty of Science and Technology, Keio University, Yokohama, Kanagawa, JAPAN

^cDepartment of Business Economics, Tokyo University of Science,

1-11-2 Fujimi, Chiyoda-ku, Tokyo 102-0071, JAPAN

Phone: +81-3-3288-2553, Fax: +81-3-3288-2498, E-mail: umezawa@rs.tus.ac.jp

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¹Corresponding author.

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Abstract

We investigate how the type of innovation, either for product or process, influences the licensing scheme. For product innovation, we consider a licensing scheme for a patent holder of a new product facing a potential rival who may invest in technology innovation and enter the market for the new product. The alternative forms of licensing schemes considered are fixed fees and royalties. We then compare this licensing scheme for the product innovation with that for the process innovation studied in the literature.

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1 Introduction

In the literature of innovation, it is customary to distinguish two types of innovation: product and process innovation, see e.g. Bonanno and Haworth (1998). The literature of licensing however focused historically mostly on the process innovation. This paper answers the following question: does the type of innovation, product and process innovation, influence the licensing form? Wang (2002) investigates the issue of fee versus royalty licensing for the case of process innovation in a differentiated Cournot duopoly. We employ the model of product innovation in a differentiated Cournot duopoly considered by Kitagawa et al. (2014) and investigate the licensing in the framework of fixed fee versus royalty licensing. Specifically, we evaluate two licensing schemes, fixed fees and royalty licensing, as tools for an incumbent innovator to license the technology for a new product to a potential competitor, who then has the option of self-developing the technology for a comparable product without patent infringement. We then compare our result with Wang (2002) and answer the aforementioned question in the framework of fee versus royalty licensing.

Early work on patent licensing can be traced back to Arrow (1962), who argued that a perfectly competitive industry provides a greater incentive to innovate than a monopoly. Kamien and Tauman (1984, 1986) and Katz and Shapiro (1985, 1986) are other early studies that analyzed licensing policies. For the most part, this early literature has mainly focused on the licensing of a cost-reducing innovation by an R&D specialized firm whose sole objective is to license the patent to other firms. One of the major findings is that fixed fee licensing is superior to royalty licensing for the patent holder (see, e.g., Kamien and Tauman 1986 and Mauleon et al. 2013). The subsequent literature has since proceeded in several different directions (see Kamien 1992 for a survey).

In some R&D environments, the innovator is one of a number of incumbent firms in the industry (see, e.g., Taylor and Silberston 1973). In such models, Gallini and Winter (1985) and Marjit (1990) consider an asymmetric cost structure of firms in a duopoly. The former analyzes a model with royalty licensing, while the latter investigates fixed fee licensing and

shows the existence of a Pareto-dominating Nash equilibrium. A large body of work also focuses on the impact of the magnitude of the cost-reduction innovation. A cost-reducing innovation is said to be drastic if the monopoly price under the new technology does not exceed the competitive price under the old technology; see Arrow (1962). This type of analysis with an incumbent innovator is conducted by Wang (1998, 2002), Faulí-Oller and Sandonís (2002), Kamien and Tauman (2002), Sen and Tauman (2007), San Martín and Saracho (2010), Li and Yanagawa (2011), and Colombo (2012). Wang (1998), for example, examines a homogenous goods duopoly model with both fixed fee and royalty licensing, and shows that royalty licensing is superior to fixed fee licensing for the patent-holding firm. Wang (2002) later extends the analysis to a differentiated goods duopoly. Kamien and Tauman (2002) extend the result of Wang (1998) to the general Cournot oligopoly with n firms, while Faulí-Oller and Sandonís (2002) consider a fixed fee plus royalty contract, and show that the optimal licensing contract always includes a royalty.

Elsewhere, Sen and Tauman (2007) consider licensing when the innovator uses combinations of fixed fee and royalty licensing in an oligopoly market of general size. Sen and Tauman (2007) analyze the case of an outside innovator as well as an incumbent innovator, and obtain the optimal licensing scheme for each case. San Martín and Saracho (2010) introduce licensing using an ad valorem royalty, and show that an incumbent innovator prefers an ad valorem royalty to a per-unit royalty. However, despite the wide coverage of this literature, the cost of technology innovation for the potential rival has not yet been investigated. An exception is Kulatilaka and Lin (2006), who first show that under a homogeneous market royalty licensing is superior to fixed fee licensing, and then construct a model for financing technology innovation. For the industry practice of licensing schemes, see Rostoker (1984) and Taylor and Silberston (1973).

Instead of assuming a cost reduction innovation, Kitagawa et al. (2014) consider a technology for a new product that can be licensed but also developed by the potential entrant to produce an imperfect substitute for the new product. They explore how the two-part tariff contract is impacted upon by the patent strength, the substitutability of

goods and the market size where the patent strength is characterized by the development cost incurred by the entrant. San Martín and Saracho (2016) investigate the same model of product innovation incorporating *ad valorem* royalties.

In this paper, to see the impact of the type of innovation on the licensing form, we investigate licensing of a product innovation in the framework of fixed fee versus royalty licensing. In the absence of licensing, the potential competitor invests in technology development to enter the market if the cost of development is small. We refer to this environment as the low development cost scenario. If this is not the case, we refer to the environment as a high development cost scenario. Note that the development cost in the product innovation has an effect similar to the magnitude of innovation in the process innovation, see Arrow (1962). Our findings are summarized as follows. 1) The licensing scheme arising in the high development cost scenario in the case of product innovation is completely identical to that in the drastic innovation in the case of process innovation. This is so because the difference between the product and process innovation lies in the case when the potential licensee chooses the competition in the production market without licensing (outside option). Under the high development cost scenario and the drastic innovation, however, this outside option is not a threat to the patent holder. 2) The low development cost scenario of the product innovation and the non-drastic innovation of the process innovation are different because the outside option is a credible threat to the patent holder and the Cournot competition under the outside option is different under the product innovation and the process innovation. 3) A large development cost (within the low development cost scenario) and a low level of substitutability tend to make a fixed fee contract superior to a royalty contract for the patent holder. Numerical experiments indicate that this tendency arises for the case of process innovation as well. We provide an intuitive argument behind it.

The remainder of the paper is organized as follows. In Section 2, we formally provide the licensing model. In Section 3, we present the main result and its interpretation. Finally, in Section 4, we present our conclusion.

2 Model

Firm 1 has a technology for a new product. Firm 2 is a potential competitor but does not have the technology. Firm 1 has an option of licensing its own technology to firm 2. We consider two types of licensing contracts, a fixed fee or a royalty. Under the fixed fee contract, firm 2 pays a lump sum $\varphi \geq 0$ independent of the amount of production. Under the royalty contract, firm 2 pays the royalty rate r per unit of production. In period 0, firm 1 decides whether to offer licensing to firm 2 and if so which type of contract. If firm 1 does not offer licensing to firm 2, firm 2 has two options in period 1. Firm 2 may stay out of the market or enter the market by developing the technology itself.

If firm 2 invests in the technology development, it incurs a cost of $J > 0$ where the development cost J corresponds to patent strength¹. We assume that the development succeeds in producing a marketable product without patent infringement. If firm 1 offers licensing, firm 2 may accept or reject the offer in period 1. In the latter case, firm 2 may remain out of the competition or enter the market by developing the technology itself. We assume that firm 2 accepts the offer if firm 2 is indifferent between accepting and rejecting the offer. For analytical convenience, we further assume that firm 1 offers the royalty contract if firm 1's profit is indifferent between the royalty contract and the fixed fee. We also assume that firm 2 enters the market if it is indifferent between entering or not entering.

In period 2, two firms engage in Cournot competition if firm 2 enters the market. Otherwise, firm 1 monopolizes the market. We assume that while the technology of the two firms may be identical, the products of the two firms may be differentiated and the demands $q_i, i = 1, 2$, for their products are characterized by the following inverse demand functions.

$$P_i = \theta - q_i - aq_j, \quad i, j = 1, 2, j \neq i, \quad (1)$$

¹Farrell and Shapiro (2008) define the patent strength by the probability that the patent would be found valid if tested in court. We do not consider uncertainty associated with litigation, and assume that the rival firm can either accept the offered ironclad license or avoid using the patented technology, and may develop the alternative technology for the new product. If the cost of self-developing the technology for a comparable product without patent infringement is high, we consider the patent to be strong.

where P_i is the price of firm i 's product². We refer to parameters $\theta > 0$ and $a \in [0, 1]$ as the market size and substitution coefficient, respectively.

For the reader's reference, Figure 1 shows the game tree where the Cournot duopoly games are abbreviated.

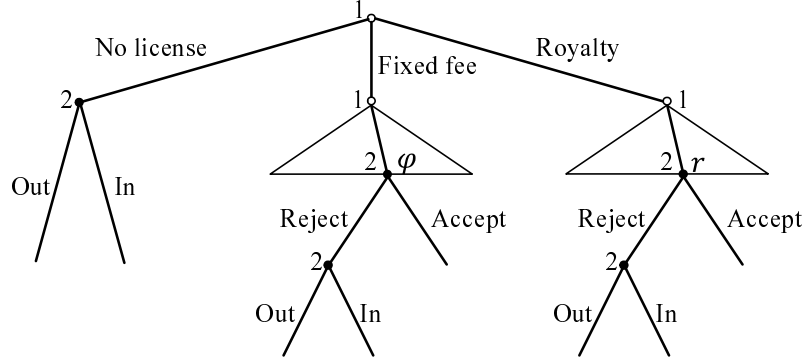


Figure 1: Game tree

3 Main Result

To derive the subgame perfect Nash equilibrium we examine the equilibrium path of each subgame.

Consider the subgame starting after firm 1 chooses not to license in period 0. If firm 2 stays out of the market, firm 1 monopolizes the market and maximizes its payoff $\Pi_1 = (\theta - q_1)q_1$ with the optimal amount of production $q_1^* = \theta/2$. The payoffs of the firms are given by $(\Pi_1, \Pi_2) = (\frac{1}{4}\theta^2, 0)$. If firm 2 enters the market, the two firms engage in Cournot competition with payoff functions $\Pi_1 = (\theta - q_1 - aq_2)q_1$ and $\Pi_2 = (\theta - q_2 - aq_1)q_2$.³ The equilibrium amount of production is

$$q_1^* = q_2^* = \frac{\theta}{a + 2},$$

²When product differentiation is firm specific, two firms using the same technology would produce differentiated varieties, see Katz (1984).

³Here we assume zero production cost. Including constant marginal production costs, however, will not change the argument, as stated in many studies along this line.

with payoff $(\Pi_1, \Pi_2) = \left(\frac{\theta^2}{(a+2)^2}, \frac{\theta^2}{(a+2)^2} - J \right)$. Let

$$\hat{J} \equiv \frac{\theta^2}{(a+2)^2}, \quad \hat{\theta} \equiv (a+2)\sqrt{J}. \quad (2)$$

When no license is offered, firm 2 enters the market if and only if the development cost J is less than or equal to \hat{J} . The development cost J corresponds to the strength of firm 1's patent because a low value of J implies that firm 2 has relatively easy access to the technology. Thus, we identify $J > (\leq) \hat{J}$ as the high (low) development cost scenario, which technically is a synonym for the low (high) demand scenario $\theta < (\geq) \hat{\theta}$. We note that although a high value for market size θ may appear to be a good sign for firm 1, it encourages firm 2 to enter the market. As we will see later, J and θ influence the form of licensing chosen by firm 1 in an opposite and peculiar way.

The difference between our model and the model of Wang (2002) is in the Cournot duopoly games after firm 2 chooses "In", see Figure 1. In our model of product innovation, in the absence of licensing, the two firms engage in a symmetric Cournot duopoly game after firm 2 invests the development cost J . In Wang's model of process innovation, in the absence of licensing, the two firms engage in an asymmetric Cournot duopoly game where firm 1 takes advantage of reduced marginal production cost due to the process innovation while firm 2 does not. Under the high development cost scenario of the product innovation and the drastic innovation case of the process innovation, by definition, strategy "In" (see Figure 1) is not a threat to firm 1, so that the product and process innovation results in the same equilibrium.

Our main results are stated in the following theorem. The high development cost scenario is same as the case of drastic innovation in Wang (2002) so that its proof is omitted. The analysis of the low development cost scenario is cumbersome and tedious so that it is relegated to the appendix.

Theorem 1 *The equilibrium is given as follows depending on the model parameters.*

1. Consider the high development cost scenario ($J > \hat{J}$). If $a \geq \bar{a}$ (region A in Figure 2) firm 1 offers a royalty contract where $\bar{a} = 0.7878$. Otherwise (region B) firm 1 offers a fixed fee contract.
2. Consider the low development cost scenario ($J \leq \hat{J}$). If ($J \leq \bar{J}_b$ and $J \leq \bar{J}$) (region C) or ($\bar{J} < J$ and $J \leq \bar{J}_t$) (region A'), firm 1 offers a royalty contract, where

$$\bar{J} = \frac{(3a^2 - 8)^2 - (2a^2 + 2a - 4)^2}{(a + 2)^2 (3a^2 - 8)^2} \theta^2,$$

$$\bar{J}_t = \frac{(a^2 - 2a - 4)^2 \theta^2}{4(a + 2)^2 (-3a^2 + 8)}$$

and \bar{J}_b is the solution of $I(J) = 0$ with

$$I(J) = \left(-\frac{3}{4}a^2 + 1\right)J + \frac{(a - 1)\theta(\theta - \sqrt{\theta^2 - (a + 2)^2 J})}{(a + 2)}.$$

Otherwise (region D) firm 1 offers a fixed fee contract.

The borderlines between regions are identified by the equations given in Table 1. Figure 2 illustrates Theorem 1. In what follows, we examine the managerial implications of Figure 2.⁴

Borderline between regions	Equation
$A \cup B$ and $A' \cup D$	$J = \hat{J}$ (or $\theta = \hat{\theta}$)
A and B	$a = \bar{a}$
C and D	$J = \bar{J}_b$
A' and C	$J = \bar{J}$
A' and D	$J = \bar{J}_t$

Table 1: Borderlines between regions

The high development cost scenario is represented by regions A (royalty) and B (fixed fee) separated horizontally. This scenario implies that the development cost J is too high for

⁴Two-part tariff licensing of product innovation with Cournot duopoly is studied by Kitagawa et al. (2014). They show that in the high development cost case with $a = 1$, firm 1 does not offer the contract. This may appear to contradict our result that the firm 1 offers the royalty contract in the aforementioned case. The result of Kitagawa et al. (2014) arises because they assume that firm 1 does not offer a contract if firm 1 is indifferent between offering and not offering one.

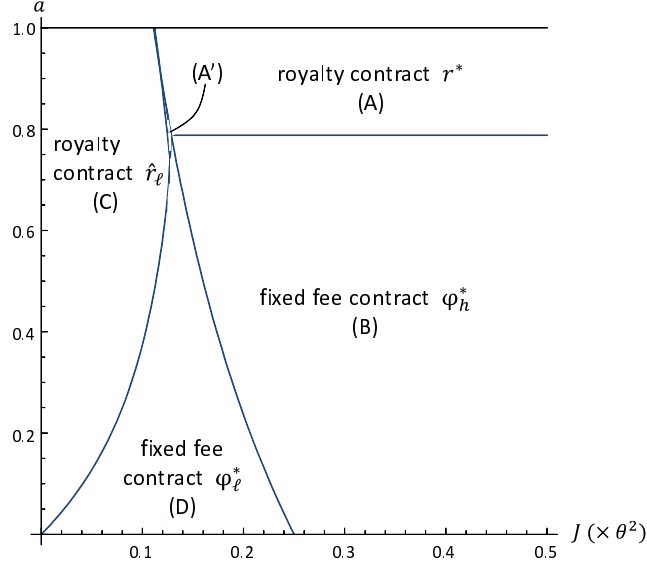


Figure 2: Type of equilibrium and model parameters (J, a)

firm 2 to enter the market by self-developing the technology. Therefore, firm 1 can design a contract without considering the threat of the self-developed product by firm 2. Thus, the magnitude of the development cost J plays no role in the choice of the optimal contract type under the high development cost scenario, so that the horizontal line arises between A and B . The licensing scheme under this scenario is identical to that under the drastic innovation of Wang (2002).

Next, we give an intuition regarding the impact of the substitution coefficient a on the choice of contract form (irrespective of the scenario). When the substitution coefficient a is close to 1, the duopolistic competition is fierce, so that firm 2 prefers a weak competitor in the production market. The royalty contract serves for this purpose since the royalty rate makes the marginal production cost of firm 2 high. On the other hand, when the substitution coefficient a is small (the two products are differentiated), firm 1 prefers that firm 2 makes a large profit in the production market so that firm 1 can extract a large license fee from firm 2 in the form of a fixed fee. Thus, irrespective of the scenario, if a is large (small), the royalty (fixed-fee) contract tends to arise.

Under the low development cost scenario, however, the optimal contract type depends

on three factors: the development cost J , market size θ , and the substitution coefficient a ; see Figure 2. It can be seen that for a development cost J sufficiently small, the royalty contract outperforms the fixed fee contract for firm 1. This is due to the fact that 1) for the Cournot duopoly model with a royalty contract, the sum of the profits of the two firms with a sufficiently small $r > 0$ is larger than that with $r = 0$; and 2) when J is small, firm 1 raises the royalty rate up to the point where firm 2 makes the same profit as when it self-develops the product, resulting in the firm 1's profit equal to the joint profit minus this fixed profit of firm 2. For a more detailed discussion with explicit formulas, readers are referred to Remark 1 in the Appendix. To see what happens when J is large within the low development cost scenario, we first note that under the high development cost scenario the fixed fee (royalty) contract arises for a small (large) substitution coefficient a , see region B (A) of Figure 2. This statement holds true even for the low development cost scenario if J is large and is close to \hat{J} since the profit function of firm 1 is *continuous* on the (J, a) plane. To summarize, under the low development cost scenario, both large a and small J tend to make the royalty contract a better choice than the fixed fee contract, see Figure 2.

The low development cost scenario is different from the non-drastic innovation because the outside option is a credible threat to the patent holder and the payoffs arising from the outside option are different in the low development cost scenario and the non-drastic innovation. However, we confirm numerically that small substitution coefficient and large magnitude of innovation tend to make the fixed fee contract a better choice than the royalty contract under the non-drastic innovation in the case of process innovation as well. Thus, the low development cost scenario and the non-drastic innovation have a similar tendency in terms of how the licensing scheme arises.^{5 6 7} The reasoning behind this involves some

⁵We note that the licensing of product innovation in a Bertrand duopoly shares the similarity that small substitution coefficient and large magnitude of innovation tend to make the fixed fee contract a better choice than the royalty contract, see Wang and Yang (1999), and Colombo (2015).

⁶In two-part tariff licensing of product innovation, Kitagawa et al. (2014) show that a pure royalty contract tends to arise when the substitution coefficient is large, and the development cost is small. Thus, the role of the substitution coefficient and the development cost in two-part tariff licensing is similar to that in fee versus royalty licensing.

⁷Wang (2002) discusses fee versus royalty licensing in the case of process innovation. In Proposition 3(i) he provides a condition under which royalty licensing is superior to fee licensing for firm 1. This

technicalities, and the readers are referred to Remark 2 in the Appendix.

Another notable feature of the product innovation is that the optimal licensing scheme depends on both the development cost (patent strength) J and market size θ but only through J/θ^2 , which implies that patent strength and market size have opposing effects on the optimal licensing scheme, see Figure 2. More specifically, multiplying θ by c has the same impact as dividing J by c^2 . We note that a similar property does not arise in the case of process innovation, see Wang (2002).

Furthermore, a weak patent, large market size, and high substitutability all tend to make the royalty contract the optimal choice for the incumbent innovator, which is consistent with the empirical finding by Vishwasrao (2007) that licensing contracts are more likely to be royalty based when sales are high.

4 Conclusion

In this paper, we examine how the type of innovation, product and process innovation, influences the licensing form in the framework of fixed fee and royalty licensing. To this end we compare these two licensing schemes as tools for an incumbent innovator to license a technology for a new product to a potential competitor, who has the option available of self-developing the technology for an imperfectly substitutable product. This outside option for the potential competitor limits the power of the incumbent innovator in extracting rents and thus affects the choice of licensing contract. We characterize the optimal licensing scheme depending on the cost of innovation J , market size θ , and the substitution coefficient a . We find that the licensing scheme under the high development cost scenario is identical to the drastic innovation in the case of process innovation while that under the low development cost is different from the non-drastic innovation.

condition contradicts our observation. Our numerical experiment shows that Proposition 3(i) does not hold for $a = 1, c = 0.8, d = 0.4, 0.2 < \varepsilon < 0.6$ in his notation.

Appendix

Let Π_i denote the payoff, with a superscript identifying a licensing policy (N, F, or R, for “no license”, “fixed fee”, or “royalty”, respectively). We obtain the following lemmas.

Lemma 2 *Consider the subgame starting after firm 1 chooses not to license.*

1. *If $J > \hat{J}$, firm 2 stays out of the market and firm 1 monopolizes the market, resulting in the payoffs*

$$(\Pi_1^N(h), \Pi_2^N(h)) \equiv \left(\frac{1}{4}\theta^2, 0 \right). \quad (3)$$

2. *If $J \leq \hat{J}$, firm 2 enters the market and the payoffs are given by*

$$(\Pi_1^N(\ell), \Pi_2^N(\ell)) \equiv \left(\frac{\theta^2}{(a+2)^2}, \frac{\theta^2}{(a+2)^2} - J \right). \quad (4)$$

Lemma 3 *Suppose $J \leq \hat{J}$. Consider the subgame starting after firm 1 chooses a fixed fee contract. Under the equilibrium path of this subgame, firm 1 offers $\varphi_\ell^* = J$, which firm 2 accepts. The payoffs are given by*

$$(\Pi_1^F(\ell), \Pi_2^F(\ell)) \equiv \left(\frac{\theta^2}{(a+2)^2} + J, \frac{\theta^2}{(a+2)^2} - J \right). \quad (5)$$

Proof. Suppose that $J \leq \hat{J}$ and firm 1 chooses to offer a fixed fee contract. The subgame starting after firm 2 rejects the offer is identical to the subgame starting after firm 1 chooses to not offer a license. Thus, from Lemma 2, if firm 2 rejects the offer, it enters the market. If firm 2 accepts the fixed fee contract, it pays φ and engages in Cournot competition with firm 1. Thus, firm 2 compares the following two payoffs when choosing whether to accept.

$$\Pi_2 = \begin{cases} \frac{\theta^2}{(a+2)^2} - \varphi & \text{if firm 2 accepts the offer,} \\ \frac{\theta^2}{(a+2)^2} - J & \text{otherwise.} \end{cases}$$

This implies that firm 2 accepts the offer if and only if firm 1 offers φ with

$$\varphi \leq \varphi_\ell^* \equiv J.$$

Hence, the firm 1's payoff is

$$\Pi_1 = \begin{cases} \frac{\theta^2}{(a+2)^2} + \varphi & \text{if firm 1 offers } \varphi \leq \varphi_\ell^* \text{ so that firm 2 accepts the offer,} \\ \frac{\theta^2}{(a+2)^2} & \text{otherwise.} \end{cases}$$

Firm 1 thus offers $\varphi = \varphi_\ell^* = J$ and firm 2 accepts it under the equilibrium path. ■

We next examine the subgame after firm 1 chooses to offer a royalty contract. When the royalty rate is too high, firm 2 does not produce after it accepts the offer. The following Lemma shows that condition

$$r > \hat{r} \equiv \frac{-a+2}{2}\theta \quad (6)$$

corresponds to this case.

Lemma 4 *Consider the subgame starting after firm 1 offers the royalty contract r and firm 2 accepts it. The payoffs of the subgame are given by*

$$(\Pi_1, \Pi_2) = \begin{cases} (\pi_1^*(r), \pi_2^*(r)) & \text{if } r \leq \hat{r}, \\ (\frac{1}{4}\theta^2, 0) & \text{if } r > \hat{r}, \end{cases} \quad (7)$$

where

$$\pi_1^*(r) = \frac{(3a^2 - 8)r^2 + (a^3 - 4a^2 + 8)r\theta + (a - 2)^2\theta^2}{(a^2 - 4)^2}, \quad (8)$$

$$\pi_2^*(r) = \frac{((-a + 2)\theta - 2r)^2}{(a^2 - 4)^2}. \quad (9)$$

Proof. In this subgame, the Cournot competition is described by

$$\begin{cases} \max_{q_1 \geq 0} \Pi_1 = (\theta - q_1 - aq_2)q_1 + rq_2, \\ \max_{q_2 \geq 0} \Pi_2 = (\theta - q_2 - aq_1 - r)q_2. \end{cases}$$

When the royalty rate is low (high), $r \leq \hat{r}$ ($r > \hat{r}$), a duopoly (a monopoly) arises with

$$(q_1^*, q_2^*) = \begin{cases} \left(\frac{\theta}{a+2} - \frac{ar}{a^2-4}, \frac{\theta}{a+2} + \frac{2r}{a^2-4} \right) & \text{if } r \leq \hat{r}, \\ \left(\frac{\theta}{2}, 0 \right) & \text{if } r > \hat{r}. \end{cases}$$

Thus, the lemma follows after some simple calculation. ■

We note that (8) is a concave quadratic function. For the sake of analytical convenience, we define

$$r^* = \arg \max_r \pi_1^*(r).$$

Specifically, r^* is given by

$$r^* = \frac{a^3 - 4a^2 + 8}{-6a^2 + 16}\theta. \quad (10)$$

We note that r^* in (10) is meaningful only when $r^* \leq \hat{r}$.

Let \hat{r}_ℓ be the royalty rate at which firm 2 is indifferent between developing the technology itself and accepting a royalty contract $r \leq \hat{r}$. More specifically, \hat{r}_ℓ is the royalty rate r at which $\Pi_2^N(\ell)$ in (4) is equal to $\Pi_2 = \pi_2^*(r)$ in (7) with (9). The royalty rate \hat{r}_ℓ is explicitly written as

$$\hat{r}_\ell \equiv \frac{-a+2}{2} \left(\theta - \sqrt{\theta^2 - (a+2)^2 \bar{J}} \right) \leq \hat{r}, \quad (11)$$

where the inequality follows from (6). Note that \hat{r}_ℓ is a well-defined real number under the low development cost scenario.

Let \bar{J} be the development cost such that

$$\hat{r}_\ell = r^*$$

holds true, see (10) and (11). Let

$$G(J) = r^* - \hat{r}_\ell.$$

Given

$$\frac{\partial G(J)}{\partial J} = \frac{(a-2)(a+2)^2}{4\sqrt{\theta^2 - (a+2)^2 J}} < 0,$$

$G(J)$ is strictly decreasing in J . After some algebra, $G(\hat{J}) \leq 0$ and $G(0) > 0$. Thus, \bar{J} is well defined. Here we note that

$$\bar{J} \leq \hat{J} \tag{12}$$

and

$$J \leq \bar{J} \Leftrightarrow \hat{r}_\ell \leq r^*. \tag{13}$$

For notational convenience, we indicate the scenarios with development cost $J \in (0, \bar{J}]$ and $J \in (\bar{J}, \hat{J}]$ by symbols ℓ_b and ℓ_t , respectively.

Lemma 5 *Suppose $J \leq \hat{J}$. Consider the subgame starting after firm 1 chooses a royalty contract. The equilibrium path of this subgame is as follows.*

1. Suppose $J \in (0, \bar{J}]$. Then, firm 1 offers \hat{r}_ℓ , and firm 2 accepts it. The payoffs are

$$\begin{aligned} \Pi_1^R(\ell_b) &\equiv \pi_1^*(\hat{r}_\ell) \\ &= \frac{-3a^2 + 8}{4}J + \frac{(a^2 + a - 1)\theta^2 - (a^2 + a - 2)\theta\sqrt{\theta^2 - (a+2)^2 J}}{(a+2)^2}, \end{aligned} \tag{14}$$

$$\Pi_2^R(\ell_b) \equiv \pi_2^*(\hat{r}_\ell) = \frac{\theta^2}{(a+2)^2} - J. \tag{15}$$

2. Suppose $J \in (\bar{J}, \hat{J}]$. Then firm 1 offers r^* in (10) and firm 2 accepts it. The payoffs

are

$$\begin{aligned}
(\Pi_1^R(\ell_t), \Pi_2^R(\ell_t)) &\equiv (\pi_1^*(r^*), \pi_2^*(r^*)) \\
&= \left(\frac{(a-6)(a-2)}{4(-3a^2+8)}\theta^2, \frac{4(a-1)^2}{(-3a^2+8)^2}\theta^2 \right). \tag{16}
\end{aligned}$$

Proof. Suppose that firm 1 chooses a royalty contract. The subgame starting after firm 2 chooses to reject the offer is the same as the subgame discussed in Lemma 2. Thus, firm 2's payoff for this subgame is $\frac{\theta^2}{(a+2)^2} - J$. Lemma 4 shows that if firm 1 offers $r > \hat{r}$ and firm 2 accepts, the payoff of firm 2 is 0. Firm 2 compares these two payoffs, finds $\frac{\theta^2}{(a+2)^2} - J \geq 0$, and thus rejects the offer. Hence from Lemma 2,

$$\Pi_1 = \frac{\theta^2}{(a+2)^2}, \quad \Pi_2 = \frac{\theta^2}{(a+2)^2} - J, \quad \text{if firm 1 offers } r > \hat{r}.$$

When firm 1 offers $r \leq \hat{r}$, firm 2's payoff is given by

$$\Pi_2 = \begin{cases} \frac{\theta^2}{(a+2)^2} - J & \text{if firm 2 rejects the offer,} \\ \pi_2^*(r) & \text{if firm 2 accepts the offer.} \end{cases}$$

See Lemmas 2 and 4. Thus, firm 2 accepts the offer if and only if $\frac{\theta^2}{(a+2)^2} - J \leq \pi_2^*(r)$. This inequality can be alternatively expressed as

$$r \leq \hat{r}_\ell.$$

Thus, from Lemmas 2 and 4, firm 1's payoff is

$$\Pi_1 = \begin{cases} \frac{\theta^2}{(a+2)^2} & \text{if firm 1 offers } r > \hat{r}_\ell \text{ so that firm 2 rejects the offer,} \\ \pi_1^*(r) & \text{if firm 1 offers } r \leq \hat{r}_\ell \text{ so that firm 2 accepts the offer.} \end{cases} \tag{17}$$

Recall that $\hat{r}_\ell \leq \hat{r}$ (see (11)) and that $\pi_1^*(r)$ is a concave quadratic function. Also note that $\pi_1^*(r) \geq \pi_1^*(0) = \frac{\theta^2}{(a+2)^2}$ for $0 \leq r \leq \min\{r^*, \hat{r}\}$. The maximization of Π_1 in (17) depends on

the relative locations of \hat{r}_ℓ and r^* . When $\hat{r}_\ell \leq r^*$, firm 1 offers \hat{r}_ℓ and firm 2 accepts it with the payoffs given by (14) and (15), see Figure 3. When $\hat{r}_\ell > r^*$, firm 1 offers r^* and firm 2 accepts it with the payoffs given by (16), see Figure 4. The lemma now follows from (13).

■

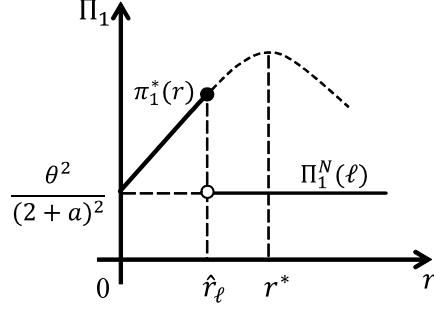


Figure 3: Firm 1's payoff when $\hat{r}_\ell \leq r^*$ (scenario ℓ_b): Since $\hat{r}_\ell \leq \hat{r}$, firm 1 chooses \hat{r}_ℓ .

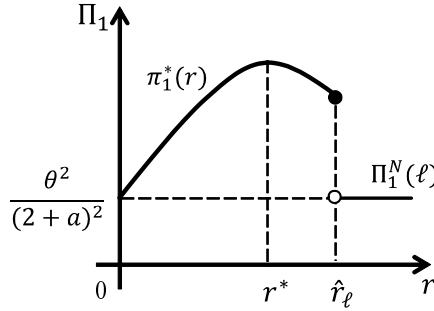


Figure 4: Firm 1's payoff when $\hat{r}_\ell > r^*$ (scenario ℓ_t): Since $r^* < \hat{r}_\ell \leq \hat{r}$, firm 1 chooses r^* .

Under the low development cost scenario, the equilibrium path depends intricately on the development cost. We need to introduce two additional thresholds for the development cost, \bar{J}_b , \bar{J}_t . Consider the case of $J \in (\bar{J}, \hat{J}]$ (scenario ℓ_t). Suppose that firm 1 compares three alternatives as in Figure 5; see Lemmas 2, 3, and 5. No license is out of consideration as it never outperforms the fixed fee licensing. Thus, firm 1 offers a royalty contract if and

only if

$$\begin{aligned}
H(J) &\equiv \Pi_1^R(\ell_t) - \Pi_1^F(\ell) \\
&= \left(\frac{(a-6)(a-2)}{4(-3a^2+8)} - \frac{1}{(a+2)^2} \right) \theta^2 - J \\
&\geq 0.
\end{aligned} \tag{18}$$

Clearly, $H(J)$ is strictly decreasing. Define \bar{J}_t by $H(\bar{J}_t) = 0$, i.e.,

$$\bar{J}_t = \frac{(a^2 - 2a - 4)^2 \theta^2}{4(a+2)^2(-3a^2+8)}.$$

We note that \bar{J}_t is well-defined since the coefficient of θ^2 in (18) is positive for $a \in [0, 1]$. We can see that if $J \leq \bar{J}_t$, firm 1 chooses a royalty contract r^* , and if $J > \bar{J}_t$, firm 1 chooses a fixed fee J .

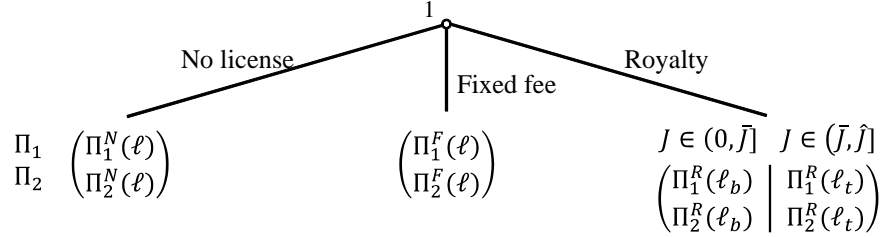


Figure 5: Reduced-game tree in low development cost scenario ($J \leq \hat{J}$)

Next consider the case of $J \in (0, \bar{J}]$ (scenario ℓ_b). Suppose that firm 1 compares a royalty contract with a fixed fee contract as above. Let

$$\begin{aligned}
I(J) &\equiv \Pi_1^R(\ell_b) - \Pi_1^F(\ell) \\
&= \left(-\frac{3}{4}a^2 + 1\right)J + \frac{(a-1)\theta(\theta - \sqrt{\theta^2 - (a+2)^2 J})}{(a+2)}.
\end{aligned}$$

We see that $I(0) = 0$. Consider the case of $0 < a \leq 1$. Then,

$$\left. \frac{\partial I(J)}{\partial J} \right|_{J=0} = -\frac{1}{4}(a-2)a > 0.$$

Moreover, for $a \in [0, 1]$,

$$\frac{\partial^2 I(J)}{\partial^2 J} = \frac{(a-1)(a+2)^3\theta}{4(\theta^2 - (a+2)^2J)\sqrt{(\theta^2 - (a+2)^2J)}} \leq 0.$$

We define \bar{J}_b by $I(\bar{J}_b) = 0$. Note that $a \in (0, -2 + 2\sqrt{2})$ if and only if $I(\hat{J}) < 0$ where $I(\hat{J}) = \frac{(a^2+4a-4)\theta^2}{4(a+2)^2}$. Thus, for $a \in (0, -2 + 2\sqrt{2})$, \bar{J}_b is well defined and $\bar{J}_b < \hat{J}$ (it may be the case that $\bar{J}_b > \bar{J}$).

We next summarize the results for the low development cost scenario ($J \leq \hat{J}$).

Lemma 6 *Consider the case of a small development cost ($J \leq \hat{J}$).*

1. *Case $J > \bar{J}$ (Scenario ℓ_t):*

- (a) *If $J \in (\bar{J}, \bar{J}_t]$, firm 1 offers a royalty contract r^* and firm 2 accepts, resulting in payoffs (16).*
- (b) *If $J \in (\bar{J}_t, \hat{J}]$, firm 1 offers a fixed fee contract $\varphi_\ell^* = J$ and firm 2 accepts, resulting in payoffs (5).*

2. *Case $J \leq \bar{J}$ (Scenario ℓ_b):*

- (a) *If $-2 + 2\sqrt{2} \leq a \leq 1$, firm 1 offers a royalty contract \hat{r}_ℓ and firm 2 accepts, resulting in payoffs (14) and (15).*
- (b) *If $0 < a < -2 + 2\sqrt{2}$ and*
 - i. *if $J \in (0, \bar{J}_b]$, firm 1 offers a royalty contract \hat{r}_ℓ and firm 2 accepts, resulting in payoffs (14) and (15).*
 - ii. *if $J \in (\bar{J}_b, \bar{J}]$, firm 1 offers a fixed fee contract $\varphi_\ell^* = J$ and firm 2 accepts, resulting in payoffs (5).*

(c) If $a = 0$, firm 1 offers a fixed fee contract $\varphi_\ell^* = J$ and firm 2 accepts, resulting in payoffs (5).

Proof. If $J \leq \hat{J}$, firm 1's payoff is given by $\Pi_1^N(\ell)$, $\Pi_1^F(\ell)$, $\Pi_1^R(\ell_t)$, or $\Pi_1^R(\ell_b)$ depending on the contract chosen and the value of J ; see Lemmas 2, 3, and 5 and Figure 5. Given $\Pi_1^F(\ell) > \Pi_1^N(\ell)$, some contract is agreed upon at the equilibrium. Consider the case of $J > \bar{J}$. Part 1 follows from the definition of \bar{J}_t . Consider Part 2(a). As stated above, $-2 + 2\sqrt{2} \leq a \leq 1$ if and only if $I(\hat{J}) \geq 0$, which implies from (12) that $I(J) \geq 0$ for $J \in (0, \bar{J}]$. Thus, firm 1 chooses a royalty contract. Thus, Part 2(a) follows from Lemma 5. Part 2(b) follows from the definition of \bar{J}_b (independent of whether or not $\bar{J}_b < \bar{J}$). If $a = 0$, we can see that $I(0) = 0$ and $\frac{\partial I(J)}{\partial J} = 1 - \frac{\theta}{\sqrt{\theta^2 - 4J}} < 0$ for $J > 0$ so that a fixed fee contract arises, completing Part 2(c). ■

We now provide the proof of our main theorem.

Proof of Theorem 1. Part 1 is identical to the case of drastic innovation in Wang (2002). Region A' corresponds to Lemma 6-1(a). Region C arises from Lemma 6-2(a) and 2(b)-i. Region D corresponds to Lemma 6-1(b), 2(b)-ii, and 2(c). ■

Remark 1 Consider the Cournot duopoly with a royalty contract. Denote the total profit of the two firms by $\pi_{tot}^*(r) = \pi_1^*(r) + \pi_2^*(r)$. Since

$$\lim_{r \rightarrow 0+} \frac{d}{dr} \pi_{tot}^*(r) = \frac{a\theta}{(a+2)^2}, \quad (19)$$

for a sufficiently small $r > 0$ and $a \neq 0$,

$$\pi_1^*(r) + \pi_2^*(r) > \pi_1^*(0) + \pi_2^*(0) = 2\pi_2^*(0)$$

holds true where $\pi_1^*(0) = \pi_2^*(0) = \frac{\theta^2}{(a+2)^2}$. For a small value of $J > 0$, the scenario of the royalty contract is given by ℓ_b . That is, firm 1 raises r up to the point where firm 2 makes the same profit as when it self-develops the product (i.e., $\Pi_2^N(\ell) = \pi_2^*(0) - J$). Under this

setting, the profit of firm 1 is

$$\Pi_1^R(\ell_b) = \pi_{tot}^*(\hat{r}_\ell) - \Pi_2^N(\ell)$$

where \hat{r}_ℓ is given by (11). Thus, when $J > 0$ is small,

$$\begin{aligned} \Pi_1^R(\ell_b) &= \pi_1^*(\hat{r}_\ell) + \pi_2^*(\hat{r}_\ell) - (\pi_2^*(0) - J) \\ &> 2\pi_2^*(0) - (\pi_2^*(0) - J) \\ &= \pi_2^*(0) + J \\ &= \pi_1^*(0) + J \\ &= \Pi_1^F(\ell). \end{aligned}$$

Hence, for $J > 0$ sufficiently small, firm 1 chooses a royalty contract.

Remark 2 Consider the case of non-drastic process innovation of Wang (2002). The reduction of marginal cost due to the process innovation is denoted by ε and is called the magnitude of innovation. Both the magnitude of innovation and the development cost J represent the strength of the patent and they play a similar role. Let $\pi_2(c_1, c_2)$ be the profit of firm 2 under Cournot duopoly with the inverse demand functions (1) and the marginal costs c_i of firm i , $i = 1, 2$. Suppose that the magnitude of innovation $\varepsilon > 0$ is sufficiently small. Under a fixed fee contract, the two firms engage in the Cournot duopoly with the marginal costs of firms 1 and 2 given by $c_1 = c - \varepsilon$ and $c_2 = c - \varepsilon$ after firm 2 pays a fixed fee $\varphi = \pi_2(c - \varepsilon, c - \varepsilon) - \pi_2(c - \varepsilon, c)$ to firm 1. Under a royalty contract, the two firms engage in the Cournot duopoly with the marginal costs of firms 1 and 2 given by $c_1 = c - \varepsilon$ and $c_2 = c - \varepsilon + r$, respectively. For $\varepsilon > 0$ small enough, the profit of firm 2 under the royalty contract is the same as its profit without licensing, see the corresponding argument in Remark 1. Also the joint profit under a royalty contract is increasing function of r for $r > 0$ sufficiently small, see (19). Thus, we can apply the same argument as Remark 1 where J is replaced by φ and see that a royalty contract arises when the magnitude of innovation $\varepsilon > 0$

is sufficiently small.

References

- Arrow, K. J. (1962), “Economic welfare and the allocation of resources for invention,” R. Nelson, ed, *The Rate and Direction of Inventive Activity*, Princeton: Princeton University Press.
- Bonanno, G. and B. Haworth (1998), “Intensity of competition and the choice between product and process innovation,” *International Journal of Industrial Organization* **16**, 495–510.
- Colombo, S. (2012), “A comment on “welfare reducing licensing”,” *Games and Economic Behavior* **76**, 515–518.
- Colombo, S. (2015), “Licensing under Bertrand competition revisited,” *Australian Economic Papers* **54**, 38–42.
- Farrell, J. and C. Shapiro (2008), “How strong are weak patents?,” *American Economic Review* **98**, 1347–1369.
- Faulí-Oller, R. and J. Sandonís (2002), “Welfare reducing licensing,” *Games and Economic Behavior* **41**, 192–295.
- Gallini, N. T. and R. A. Winter (1985), “Licensing in the theory of innovation,” *RAND Journal of Economics* **16**, 237–252.
- Kamien, M. I. (1992), “Patent licensing,” R. J. Aumann and S. Hart, eds, *Handbook of Game Theory with Economic Applications*, 331–354, Amsterdam: Elsevier Science/North-Holland.
- Kamien, M. I. and Y. Tauman (1984), “The private value of a patent: A game theoretic analysis,” *Journal of Economics (Supplement)* **4**, 93–118.
- Kamien, M. I. and Y. Tauman (1986), “Fees versus royalties and the private value of a patent,” *Quarterly Journal of Economics* **101**, 471–491.

- Kamien, M. I. and Y. Tauman (2002), “Patent licensing: The inside story,” *The Manchester School* **70**, 7–15.
- Katz, M. L. (1984), “Firm-specific differentiation and competition among multiproduct firms,” *Journal of Business* **57**, S149-S166.
- Katz, M. L. and C. Shapiro (1985). “On the licensing of innovations,” *RAND Journal of Economics* **16**, 504-520.
- Katz, M. L. and C. Shapiro (1986), “How to license intangible property,” *Quarterly Journal of Economics* **101**, 567–589.
- Kitagawa, T., Y. Masuda, and M. Umezawa (2014), “Patent strength and optimal two-part tariff licensing with a potential rival,” *Economics Letters* **123**, 227–231.
- Kulatilaka, N. and L. Lin (2006), “Impact of licensing on investment and financing of technology development,” *Management Science* **52**, 1824–1837.
- Li, Y. and T. Yanagawa (2011), “Patent licensing of Stackelberg manufacturer in a differentiated product market,” *International Journal of Economic Theory* **7**, 7–20.
- Marjit, S. (1990), “On a non-cooperative theory of technology transfer,” *Economics Letters* **33**, 293–298.
- Mauleon, A., V. Vannetelbosch, and C. Vergari (2013), “Bargaining and delay in patent licensing,” *International Journal of Economic Theory* **9**, 279–302.
- Rostoker, M. (1984), “A survey of corporate licensing,” *IDEA: The Journal of Law and Technology* **24**, 59–92.
- San Martín, M. and A. I. Saracho (2010), “Royalty licensing,” *Economics Letters* **107**, 284–287.

- San Martín, M. and A. I. Saracho (2016), “Patent strength and optimal two-part tariff licensing with a potential rival incorporating ad valorem royalties,” *Economics Letters* **143**, 28–31.
- Saracho, A. I. (2002), “Patent licensing under strategic delegation,” *Journal of Economics and Management Strategy* **11**, 225–251.
- Sen, D. and Y. Tauman (2007), “General licensing schemes for a cost-reducing innovation,” *Games and Economic Behavior* **59**, 163–186.
- Taylor, C. and Z. Silberston (1973), *The Economic Impact of the Patent System*, Cambridge University Press.
- Vishwasrao, S. (2007), “Royalties vs. fees: How do firms pay for foreign technology?,” *International Journal of Industrial Organization* **25**, 741–759.
- Wang, X. H. (1998), “Fee versus royalty licensing in a Cournot duopoly model,” *Economics Letters* **60**, 55–62.
- Wang, X. H. (2002), “Fee versus royalty licensing in a differentiated Cournot duopoly,” *Journal of Economics and Business* **54**, 253–266.
- Wang, X. H. and B. Z. Yang (1999), “On licensing under Bertrand competition,” *Australian Economic Papers* **38**, 106–119.