Corrigendum to "Behavior-based Price Discrimination in a Horizontally and Vertically Differentiated Duopoly with Switching Costs"

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January 26, 2023

Abstract

This note corrects errors in Umezawa (2022).

Umezawa (2022) studies behavior-based price discrimination (BBPD) in a horizontally and vertically differentiated duopoly with switching costs. In Section 3.1 of the article, the case of uniform pricing is considered as a benchmark. In the second period of the game, two cases of (a) $p_A^{U2} \geq p_B^{U2}$ and (b) $p_A^{U2} \leq p_B^{U2}$ are separately examined. Then, it is claimed that in the case of (a) $p_A^{U2} \geq p_B^{U2}$ ((b) $p_A^{U2} \leq p_B^{U2}$), there are no switchers that buy from firm B (A) in the first period and from firm A (B) in the second period (i.e., $z^{U2} \leq z^{U1}$ ($z^{U2} \geq z^{U1}$)). This claim, however, is not correct. That is, $p_A^{U2} \geq p_B^{U2}$ does not necessarily imply $z^{U2} \leq z^{U1}$. Similarly, $p_A^{U2} \leq p_B^{U2}$ does not necessarily imply $z^{U2} \geq z^{U1}$. Therefore, these two cases should be set up as (a) $z^{U2} \leq z^{U1}$ and (b) $z^{U2} \geq z^{U1}$, instead of (a) $p_A^{U2} \geq p_B^{U2}$ and (b) $p_A^{U2} \leq p_B^{U2}$.[1]

Accordingly, the constraint (9) of parameters is removed, and some figures in the article are modified, where each E of the areas that are examined in the models is expanded (see Figures 1, 2, 3, 4, and 5). Moreover, given these corrections, statements of some propositions in Umezawa (2022) will naturally be modified as follows. Note that with these modifications, the main conclusions in Umezawa (2022) remain unchanged.

Proposition 5 (iii) *Firm A's total profit is lower under BBPD than under UHb. Firm B's total profit is higher under BBPD than under UHb if* $s < \sigma_B^{IIHb}$, *where* $\sigma_B^{IIHb} = \frac{87N_A^2 - 142N_AN_B - 201N_B^2}{512(3N_A+5N_B)}$, *while it is not higher otherwise (see Figure 1 of this note*[2]*).*[3]

*I gratefully acknowledge the financial support of the Japan Society for the Promotion of Science through KAKENHI Grant No. 20K01619. Any errors remaining are mine.

[1] Note that the equilibria under uniform pricing are separately investigated under each of the conditions (a) and (b).

[2] The dotted curve σ_B^{IIHb} is added to the original figure.

[3] It is evident from the fact that $\pi_B^{II} - \pi_B^{UHb} = \frac{1}{2304(N_A+N_B)}[87N_A^2 - 142N_AN_B - 201N_B^2 - 512(3N_A+5N_B)s]$.

This is a similar result to Proposition 5-(i), where the firm profits under BBPD are compared with those under UHa, instead of UHb.

Proposition 6 (iv) *Consider only the cases satisfying (11). Each firm's total profit is lower under BBPD than that under ULb.*

Proposition 7 (iv) *CS is lower under BBPD than under ULa, where we consider only the cases satisfying (7). CS is higher under BBPD than under ULb, where we consider only the cases satisfying (11).*

Proposition 9 (i) *Consider case (II). Firm A's total profit is higher under BBPD than under UHb if* $s > \tau_A^{IIHb}$ *and than under ULb if* $s > \tau_A^{IILb}$*, where* $\tau_A^{IIHb} = \frac{1}{42}(7N_B + \sqrt{36N_A^2 + 1776N_AN_B + 589N_B^2})$ *and* $\tau_A^{IILb} = \frac{3N_A^2+148N_AN_B+45N_B^2}{98(3N_A+N_B)}$*. Firm B's total profit is higher under BBPD than under ULa if* $s > \tau_B^{IILa}$*. SW is higher under BBPD than under each of UHb and ULb if* $s > \tau_{SW}^{IIb}$ *(see Figure 3 of this note*[4]*).*

I would like to apologize for any inconvenience caused.

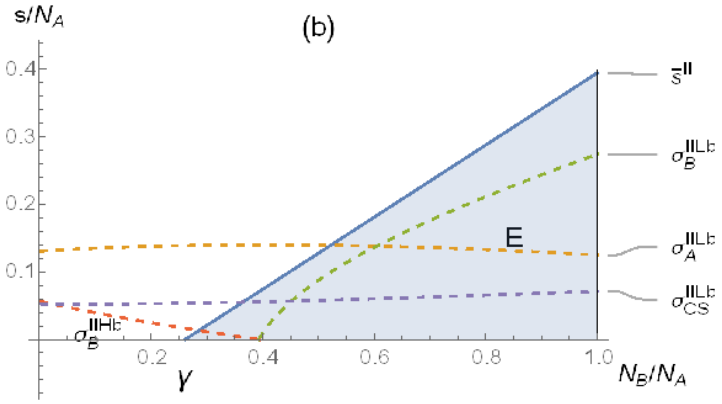

Figure 1: Correction of Figure 5 (b) (Comparison of firm profits and CS between BBPD in case (II) and UP)

References

[1] Umezawa, M., 2022. Behavior-based Price Discrimination in a Horizontally and Vertically Differentiated Duopoly with Switching Costs, Information Economics and Policy. 61, 101004. https://doi.org/10.1016/j.infoecopol.2022.101004.

[4] The dotted curve τ_A^{IIHb} is added to the original figure.

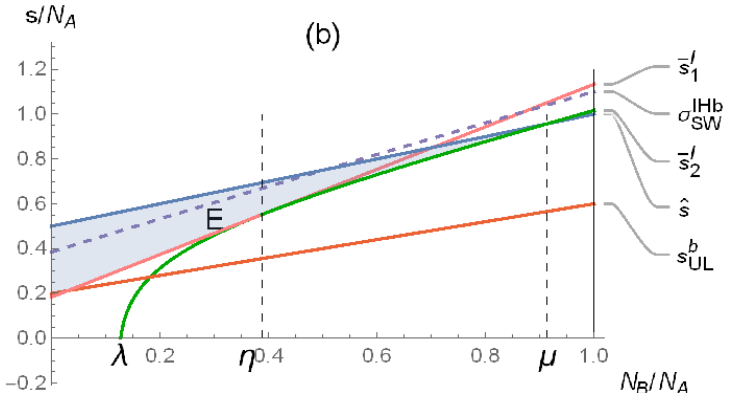

Figure 2: Correction of Figure 6 (b) (Comparison of firm profits, CS, and SW between BBPD in case (I) and UP)

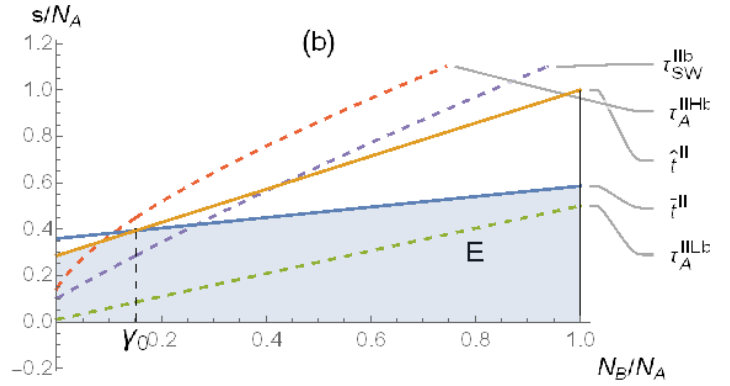

Figure 3: Correction of Figure 7 (b) (Comparison of firm profits, CS, and SW between BBPD in case (II) and UP when $\delta = 0$)

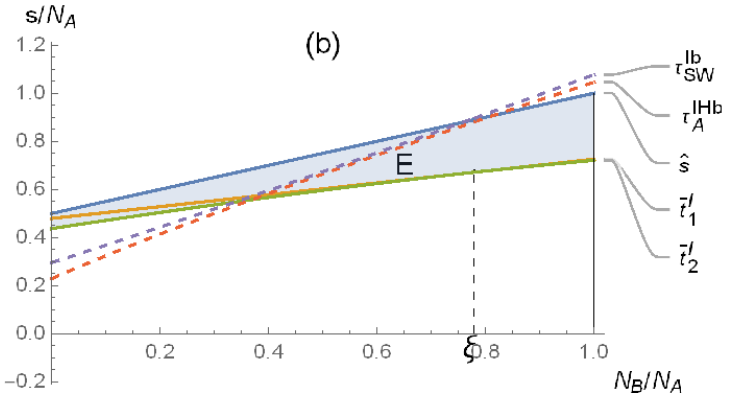

Figure 4: Correction of Figure 8 (b) (Comparison of firm profits, CS, and SW between BBPD in case (I) and UP when $\delta = 0$)

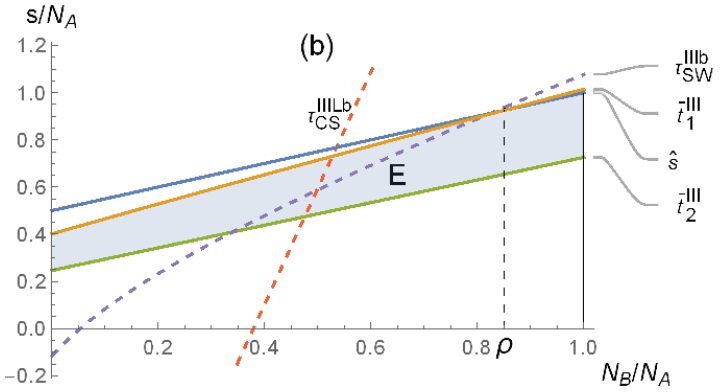

Figure 5: Correction of Figure 9 (b) (Comparison of firm profits, CS, and SW between BBPD in case (III) and UP when $\delta = 0$)

Appendix C. Supplemental Online Appendix

Proof of Proposition 6 (iv)

We have $\pi_A^I - \pi_A^{ULb} = \frac{1}{6084(N_A+N_B)}(-1373N_A^2 - 2462N_AN_B - 665N_B^2 + (4640N_A + 1528N_B)s + 28s^2)$, which is a convex quadratic function of s that takes a minimum value for some negative value of s. Since $s \leq s_{UL}^b$, $\pi_A^I - \pi_A^{ULb} \leq \frac{-137(3N_A+N_B)^2}{6084(N_A+N_B)} < 0$.

Similarly, $\pi_B^I - \pi_B^{ULb} = \frac{1}{6084(N_A+N_B)}(-227N_A^2 - 1658N_AN_B - 1607N_B^2 + (152N_A + 2056N_B)s + 3700s^2) \leq \frac{-(27N_A^2+330N_AN_B+107N_B^2)}{3380(N_A+N_B)} < 0$.

Proof of Proposition 7 (iv)

As for the proof of $CS^I - CS^{ULa} < 0$, see Umezawa (2022). We have $CS^I - CS^{ULb} = \frac{1}{6084(N_A+N_B)}(1151N_A^2 + 3308N_AN_B + 2579N_B^2 - (4970N_A + 6394N_B)s - 226s^2)$, which is a concave quadratic function of s. Let s_7 and s_8 ($s_7 < s_8$) be the solutions of $CS^I - CS^{ULb} = 0$ for s. Clearly, $s_7 < 0$. It is confirmed that $s_8 - s_{UL}^b > 0$. Thus, $CS^I - CS^{ULb} > 0$.